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Interference Alignment Configurations for Distributed Crosstalk Cancellation in G.fast

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Abstract :
In this paper, we evaluate various system configurations for crosstalk cancellation using interference alignment (IA) in uncoordinated and partially coordinated downstream G. fast. In the first configuration, which can be implemented in a distributed fashion, signal-level coordination between users is unnecessary and IA is applied in the frequency domain. In the second configuration, partial signal-level coordination is available; however, it is entirely consumed by vectoring and IA is applied in the frequency domain. In the third configuration, partial signal-level coordination is available, but vectoring is not applied. Instead, IA is applied jointly in both the space and frequency domains. Considerably higher sum bit rates are achieved using the third configuration; however, the computational complexity is noticeably higher than the other two configurations. To tackle this problem, we propose a joint partial vectoring design based on the third configuration which outperforms the diagonalizing pre-compensator (a.k.a. zero-forcing precoder) considerably. Then we employ it in the second configuration to obtain a high-performance crosstalk cancellation scheme with much lower complexity. Our simulation results show that depending on the level of coordination, IA increases the achievable rates of G. fast loops significantly compared to available solutions with comparable complexity.

Keywords: Digital Subscriber Line (DSL), Dynamic Spectrum Management, Fast Access to Subscriber Terminal (G. fast), Interference Alignment, Interfering Broadcast Channels, Vectoring.

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1. Introduction

Digital subscriber line (DSL) systems use existing telephone cable infrastructure to provide cost-effective broadband services to homes and small businesses. One of the main sources of performance degradation in advanced DSL systems is crosstalk, i.e., the undesired signal of an unintended DSL transmitter being received by the intended DSL receiver due to the electromagnetic induction between the twisted-wire pairs (TP) inside a telephone cable. Far-end crosstalk couplings between TP's in telephone cables scale quadratically by frequency. Fast access to subscriber terminals (G.fast) is a DSL protocol standard designed for local loops under 500 meters, aiming to deliver speeds ranging from 100Mbit/s to 1Gbit/s depending on the loop's length [1]. G.fast supports 106 and 212 MHz power spectral density (PSD) profiles, meaning that the transmission bandwidth is increased substantially compared to the earlier DSL standards including VDSL2. Consequently, the crosstalk couplings are very strong for G.fast and the crosstalk power between the loops could easily cease transmission in most of the available bandwidth. Crosstalk mitigation schemes are inevitable for efficient bandwidth usage in G.fast systems.

Vectoring is an effective approach for eliminating crosstalk in DSL and is mandatory in G.fast [1]. Vectoring can be implemented in DSL when all DSL lines are co-located at least at one end (usually at the central office (CO) or a street cabinet). Vectored DSL precoding and decoding techniques are applied at the transmitter and receiver sides for crosstalk mitigation in downstream (DS) and upstream (US) directions, respectively. Vectoring increases achievable data rates significantly, but it has some challenges in practical implementations [2]. Vectored DSL requires perfect signal coordination between users [3]. When the number of users is large or the DSL lines are not connected to the backbone network at the same location (e.g. as in the central office/ remote terminal (CO/RT) deployments), vectoring cannot be applied to all users. Moreover, when loop unbundling policies are in effect, it is not easy for rival companies sharing a cable to implement full vectoring because of logistical, business-related, and customer privacy considerations.

To resolve this problem, users can be divided into a few groups. Crosstalk between the users in the same group (intra-group crosstalk) can be eliminated using vectoring; however, crosstalk between users in different groups (intergroup crosstalk) remains. In [4] and [5], joint dynamic spectrum management levels 2 and 3 (DSM 2/3) have been proposed to mitigate the intra-group crosstalk and inter-group crosstalk simultaneously. It has been shown that joint DSM 2/3 is capable of canceling crosstalk in VDSL2 systems effectively [4],[5]. Unfortunately, when the number of users is more than five, the computational complexity of these techniques can be prohibitive, particularly in the DS direction, where the optimal precoder structure in each group depends on the precoder structures in other groups.

Interference alignment (IA) has been proposed for intergroup crosstalk mitigation [6]. IA has two main advantages over vectoring. The first advantage is that it does not require signal-level coordination between users [8]. However, as we will see later, it can benefit significantly from (partial) signal-level coordination between them. Therefore, it can be implemented in scenarios where users are connected to the backbone network at different locations or when different vendors share a cable. Secondly, IA can be implemented in a distributed fashion which makes it much easier to implement and much more scalable. Moreover, IA requires channel state information at the transmitter (CSIT) [22]. The DSL channel is deterministic and constant over long periods. This makes IA implementation on DSL much easier compared to on wireless cellular systems, where the channel varies over time much faster potentially resulting in significant CSIT overhead.

IA is usually considered in multiple-input multiple-output (MIMO) wireless systems where each user is equipped with multiple antennas at both the transmitter and the receiver sides. In DSL systems, the users are usually connected to a single TP. Therefore, there exists only one space dimension per user which cannot be divided into two non-empty signal and interference sub-spaces. However, IA can be implemented on other signal dimensions such as time and frequency [8]. G.fast uses discrete multi-tone (DMT) modulation [1]. Therefore, the frequency dimension is the main candidate for implementing IA in G.fast. Moreover, when users are partially coordinated in a few groups, each group is connected to several TPs. In these scenarios, IA can be implemented over the space dimension as well where each group plays the role of a single user transmitting several independent data streams to the corresponding users. Such a scenario is called the interfering broadcast (IF-BC) channel [5].

The combination of partial vectoring (PV) and IA has been considered for VDSL2 in [6], where the diagonalizing pre-compensator (DP)[7] is used in each group to cancel the intra-group crosstalk and IA is applied in the frequency dimension to mitigate the inter-group crosstalk. In this paper, we consider IA for G.fast in three system configurations:

In the first configuration, signal-level coordination is not considered between users, and IA is applied by each user independently over the frequency dimension only. This may seem to result in a poor bit rate performance as vectoring is not used at all. However, the theory behind IA guarantees that the users can asymptotically achieve half of their crosstalk-free channel bit rates [8]. Available schemes for crosstalk mitigation when signal-level coordination is not available between users such as iterative water filling [9] and optimal spectrum balancing [10], are based on optimizing the transmit spectrum of users jointly or autonomously to minimize the crosstalk power. However, this approach is highly ineffective in G.fast due to the excessive crosstalk power levels. In the second and third configurations, we consider partial signal coordination between users. In the second configuration, we consider the system proposed in [6], i.e., we use the DP in each group and then we apply IA merely over the space dimension for each user. In the third configuration, however, we do not use vectoring. Instead, we model the entire system as an IFBC channel and apply IA on space and frequency dimensions jointly.

Our simulation results show that IA can increase the achievable rates of the users significantly depending on the degree of coordination between users. Particularly, the achievable rates by IA in configuration 1, where there is no signal coordination between users, is crucial. IA achieves the best performance in configuration 3. However, the computational complexity of IA in this configuration is considerably higher than that of PV and the first two configurations.

To resolve this problem, we propose two new sub-configurations. We note that the DP does not perform very well in partially coordinated DSL systems. This is because it cancels the intra-group crosstalk perfectly without reducing the inter-group crosstalk. In [5], it has been shown that much higher bit rates can be achieved if the precoders in different groups are designed jointly. In this paper, we show that the third configuration can be used to solve the joint precoder design problem by applying IA merely on the space dimension. This solution, called sub-configuration 3, is practically appealing because it does not impose extra complexity as vectoring is already supported in G.fast. Moreover, by replacing the DP with the resulting vectoring scheme in configuration 2, we obtain a new scheme with the same computational complexity as the second configuration which nearly achieves the same sum rates as configuration 3.

Finding the ideal IA solution is usually impossible and closed-form solutions exist only in a few particular cases. However, there are a few iterative algorithms which solve the problem approximately. In [11], two algorithms have been introduced for IA in wireless channels, namely, the minimum interference leakage (minIL) algorithm and the maximum SINR (maxSINR). The maxSum-Rate algorithm also has been introduced in [12]. The performance of these algorithms in G.fast networks is evaluated and compared in this paper using computer simulations.

The paper is organized as follows. The DSL system model is described in Sec. 2. The concept of IA, the proposed configurations, how the precoder and decoder matrices are calculated, and the computational complexity of IA are discussed in Sec. 3. Simulation results are presented in Sec. 4. Finally, the results are concluded in Sec. 5.

1. System Model

We consider a DSL system with K tones and N users. We assume that the users are divided into G groups, where $\mathcal{N}_i$ and $N_i = |\mathcal{N}_i|, i = 1, \ldots, G$ denote the set and the number of users in group i, respectively. We denote the group that an arbitrary user n belongs to by $\mathcal{G}_n$, i.e., $n \in \mathcal{N}_i \Leftrightarrow \mathcal{G}_n = i$. We assume that only the users in the same group have signal-level coordination with each other. We assume that all users use synchronous DMT transmission. Under this assumption, the transmitted signal on tone $k, k = 1, \ldots, K$, is written by [13]:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k \tag{1}$$

where

$\mathbf{y}_k = \left[\left(\mathbf{y}_k^{(1)}\right)^{\mathrm{T}}, \ldots, \left(\mathbf{y}_k^{(G)}\right)^{\mathrm{T}}\right]^{\mathrm{T}} = \left[y_k^{(1)}, \ldots, y_k^{(N)}\right]^{\mathrm{T}}$,

$\mathbf{x}_k = \left[\left(\mathbf{x}_k^{(1)}\right)^{\mathrm{T}}, \ldots, \left(\mathbf{x}_k^{(G)}\right)^{\mathrm{T}}\right]^{\mathrm{T}} = \left[x_k^{(1)}, \ldots, x_k^{(N)}\right]^{\mathrm{T}}$, and

$\mathbf{z}_k = \left[\left(\mathbf{z}_k^{(1)}\right)^{\mathrm{T}}, \ldots, \left(\mathbf{z}_k^{(G)}\right)^{\mathrm{T}}\right]^{\mathrm{T}} = \left[z_k^{(1)}, \ldots, z_k^{(N)}\right]^{\mathrm{T}}$ are the vectors of the received, transmitted and Gaussian noise on tone k, respectively, and $(\cdot)^{\mathrm{T}}$ denotes the vector and matrix transpose operation. Parameters $\mathbf{y}_k^{(i)}, \mathbf{x}_k^{(i)}$, and $\mathbf{z}_k^{(i)}$ denote the vectors of the received, transmitted and Gaussian noise signal for group i and $y_k^{(n)}, x_k^{(n)}$, and $z_k^{(n)}$ denote the received, transmitted and noise signals of user n on tone k, respectively. The transmit power spectral density (PSD) for user n on tone k is denoted by $p_k^{(n)} = \frac{\mathcal{E}\left[\left|x_k^{(n)}\right|^2\right]}{\Delta f}$, where $\mathcal{E}[\cdot]$ denotes the expected value and Δf denotes the tone spacing. Finally, $\sigma_k^{(n)} = \frac{\mathcal{E}\left[\left|z_k^{(n)}\right|^2\right]}{\Delta f}$ denotes the noise PSD for user nn on tone k. The channel matrix on tone $k, \mathbf{H}_k$, is $N \times N$ and can be written as:

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{H}_k^{11} & \mathbf{H}_k^{12} & \cdots & \mathbf{H}_k^{1G} \\ \mathbf{H}_k^{21} & \mathbf{H}_k^{22} & \cdots & \mathbf{H}_k^{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_k^{G1} & \mathbf{H}_k^{G2} & \cdots & \mathbf{H}_k^{GG} \end{bmatrix} \tag{2}$$

where $h_k^{(n,m)} \equiv [\mathbf{H}_k]_{n,m}$ is the channel response from the m-th user to the n-th user on frequency tone k. The submatrix $\mathbf{H}_k^{ij}$ for $i \neq j$ denotes the crosstalk channel from group j to group i and $\mathbf{H}_k^{ii}$ denotes the direct and crosstalk channel matrix in group i. In this paper, we consider downstream (DS) transmission. However, the technique can be applied to upstream (US) transmission as well. We assume that all submatrices $\mathbf{H}_k^{ij} (1 \leq i, j \leq G)$ are perfectly known and can be used for direct channel and crosstalk calculations.

1.1. Systems with Partial Signal Coordination Using Vectoring

In this section, we present formulas for calculating the bit rate for conventional systems using PV. The bit rate calculations for systems using IA are different from these systems and will be discussed in Sec. 3.

In this paper, we consider the diagonalizing precoder [7] as the crosstalk cancellation scheme. The diagonalizing precoder multiplies the transmitted signal vector in each group i by a precoding matrix $\mathbf{B}_k^{(i)} = \left(\beta_k^{(i)}\right)^{-1} \left(\mathbf{H}_k^{ii}\right)^{-1} \mathrm{diag}\{\mathbf{H}_k^{ii}\}$, where $\mathrm{diag}(\mathbf{C})$ denotes a diagonal matrix with diagonal elements the same as those of $\mathbf{C}$ and $\beta_k^{(i)} = \max_{m;1 \leq m \leq N_i} \left\|\left[\left(\mathbf{H}_k^{ii}\right)^{-1} \mathrm{diag}\{\mathbf{H}_k^{ii}\}\right]_{m\star}\right|$, where $\mathbf{C}_{m\star}$ and $|\mathbf{v}|$denote the m-th row of matrix $\mathbf{C}$ and the Euclidean norm of vector $\mathbf{v}$, respectively [7].

From (1) and (2), the received signal vector in group ii is obtained by:

$$\mathbf{y}_k^{(i)} = \mathbf{H}_k^{ii}\mathbf{B}_k^{(i)}\mathbf{x}_k^{(i)} + \sum_{j=1;j\neq i}^{G} \mathbf{H}_k^{ij}\mathbf{B}_k^{(j)}\mathbf{x}_k^{(j)} + \mathbf{z}_k^{(i)}. \tag{3}$$

With this notation, we can rewrite the combination of the group precoders and the channel matrix as follows:

$$\overline{\mathbf{H}}_k = \begin{bmatrix} \overline{\mathbf{H}}_k^{11} & \overline{\mathbf{H}}_k^{12} & \cdots & \overline{\mathbf{H}}_k^{1G} \\ \overline{\mathbf{H}}_k^{21} & \overline{\mathbf{H}}_k^{22} & \cdots & \overline{\mathbf{H}}_k^{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\mathbf{H}}_k^{G1} & \overline{\mathbf{H}}_k^{G2} & \cdots & \overline{\mathbf{H}}_k^{GG} \end{bmatrix} \tag{4}$$

where $\overline{\mathrm{H}}_\mathrm{k}^\mathrm{ii} = \left(\beta_\mathrm{k}^\mathrm{i}\right)^{-1}\mathrm{diag}\{\mathrm{H}_\mathrm{k}^\mathrm{ii}\}$ and $\overline{\mathrm{H}}_\mathrm{k}^\mathrm{ij} = \mathrm{H}_\mathrm{k}^\mathrm{ij}\mathrm{B}_\mathrm{k}^\mathrm{j}, \mathrm{i} \neq \mathrm{j}$ are the modified intra-group crosstalk channels after PV.

In DSL, the number of bits that can be transmitted on tone k for user n is obtained as follows:

$$b_k^{(n)} = \log_2\left(1 + \frac{1}{\Gamma}\mathrm{SINR}_k^{(n)}\right) \tag{5}$$

where $\mathrm{SINR}_k^{(n)}$ denotes the signal-to-interference plus noise ratio (SINR) of user n on frequency tone k and Γ denotes the SNR gap which is a function of the bit error rate. In practical DSL systems, $b_k^{(n)}$ is truncated to an integer number and is bounded by $b_{\max}$ where $2^{b_{\max}}$ is the maximum QAM constellation size.

The SINR of user nn on frequency tone k is given by:

$$\mathrm{SINR}_k^{(n)} = \frac{\left|\bar{h}_k^{(n,n)}\right|^2 p_k^{(n)}}{\sum_{m\in\mathcal{J}_n}\left|\bar{h}_k^{(n,m)}\right|^2 p_k^{(m)} + \sigma_k^{(n}} \tag{6}$$

where $\mathcal{J}_n = \{1,\dots,N\} \setminus \mathcal{N}_i$, for $i = \mathcal{G}_n$, is the set of interfering users for user n, $\setminus$ denotes set subtraction operation and $\bar{h}_k^{(n,m)} = [\overline{\mathbf{H}}_k]_{n,m}$ denotes the crosstalk channel gain from user m to user n on tone k. The SINR for non-vectored DSL systems is obtained by setting $\mathcal{J}_n = \{1,\dots,n-1,n+1,\dots,N\}$ and replacing $\bar{h}_k^{(n,n)}$ and $\bar{h}_k^{(n,m)}$ with $h_k^{(n,n)}$ and $h_k^{(n,m)}$ in (6), respectively.

Finally, the sum-rate of user n is given by:

$$R^{(n)} = X_{\mathrm{frame}}X_{\mathrm{TDD}}f_s\sum_{k=1}^{K} b_k^{(n)} \tag{7}$$

where f_s denotes the DMT symbol rate and following [14] we ignore X_{frame} and X_{TDD} for simplicity.

2. Interference Alignment Configurations

In the following subsections, we explain the transmitter and receiver structures for several system configurations for applying IA in DS G.fast.

2.1. First and Second Configurations

In the first Configuration, G.fast users are not coordinated at the signal level. In the second Configuration G.fast users enjoy partial signal level coordination; however, it has been consumed entirely by vectoring using the diagonalizing precoder (DP) inside groups as explained in Sec. 2.1. Therefore, in both configurations, each user has access to a single spatial dimension but K frequency dimensions corresponding to the DMT tones at both the transmitter and the receiver sides. In either case, IA is implemented almost the same as in the MIMO interference channel, where the DMT tones play the role of transmitter and receiver antennas.

The interference is aligned by using precoder and decoder matrices applied on $\tilde{\mathbf{x}}^{(n)} = \left[\tilde{x}_1^{(n)},\dots,\tilde{x}_{d_n}^{(n)}\right]^{\mathrm{T}}$, where $d_n \leq K$ is the number of data streams for user n. In the first and second configurations, the channel matrix from transmitter m to receiver n is $\mathbf{H}^{(n,m)} = \mathrm{diag}\left\{h_1^{(n,m)},\dots,h_K^{(n,m)}\right\}$ and $\overline{\mathbf{H}}^{(n,m)} = \mathrm{diag}\left\{\bar{h}_1^{(n,m)},\dots,\bar{h}_K^{(n,m)}\right\}$, respectively, where $h_k^{(n,m)}$ and $\bar{h}_k^{(n,m)}$ are defined in Sec. 2. The off-diagonal elements of $\mathbf{H}^{(n,m)}$ are zero, because the DMT tones are orthogonal. Moreover, in the second configuration, the diagonal elements of $\mathbf{H}^{(n,m)}$ are zero whenever users

m and n ($m \neq n$) belong to the same group thanks to the DP.

The recovered data vector for user n is:

$$\begin{aligned}\tilde{\mathbf{y}}^{(n)} &= \left[\mathbf{U}^{(n)}\right]^{\mathrm{H}}\mathbf{H}^{(n,n)}\mathbf{V}^{(n)}\tilde{\mathbf{x}}^{(n)} \\ &+ \sum_{m\in\mathcal{I}_n} \left[\mathbf{U}^{(n)}\right]^{\mathrm{H}}\mathbf{H}^{(n,m)}\mathbf{V}^{(m)}\tilde{\mathbf{x}}^{(m)} + \tilde{\mathbf{z}}^{(n)}\end{aligned} \tag{8}$$

where $\mathbf{U}^{(n)}$and $\mathbf{V}^{(n)}$ are the decoder and precoder matrices for user $n, \tilde{\mathbf{z}}^{(n)} = \left[\mathbf{U}^{(n)}\right]^{\mathrm{H}}\mathbf{z}^{(n)}$, and $\mathcal{I}_n$ is equal to $\{1, \dots, n-1, n+1, \dots, N\}$ and $\{1, \dots, N\} \setminus \mathcal{N}_i$, where $i = \mathcal{G}_n$, for the first and second configurations, respectively.

The number of bits that can be transmitted on the d-th stream of user n per DMT symbol is calculated by:

$$\tilde{\mathrm{b}}_{\mathrm{d}}^{(\mathrm{n})} = \log_2\left(1 + \frac{1}{\Gamma}\mathrm{SINR}_{\mathrm{d}}^{(\mathrm{n})}\right) \tag{9}$$

where

$$\mathrm{SINR}_{\mathrm{d}}^{(\mathrm{n})} = \frac{\left[\mathbf{U}_{\star\mathrm{d}}^{(\mathrm{n})}\right]^{\mathrm{H}}\mathbf{H}^{(\mathrm{n,n})}\mathbf{V}_{\star\mathrm{d}}^{(\mathrm{n})}\left[\mathbf{V}_{\star\mathrm{d}}^{(\mathrm{n})\ \mathrm{H}}\left[\mathbf{H}^{(\mathrm{n,n})}\right]^{\mathrm{H}}\mathbf{U}_{\star\mathrm{d}}^{(\mathrm{n})}\mathrm{p}_{\mathrm{d}}^{(\mathrm{n})}}{\left[\mathbf{U}_{\star\mathrm{d}}^{(\mathrm{n})}\right]^{\mathrm{H}}\mathbf{X}^{(\mathrm{n})}\mathbf{U}_{\star\mathrm{d}}^{(\mathrm{n})}} \tag{10}$$

denotes the SINR for the d th stream of user n

$$\mathbf{X}^{(\mathrm{n})} \equiv \sigma_{\mathrm{n}}^{2}\mathbf{I}_{\mathrm{K}} + \mathbf{Q}^{(\mathrm{n})} \tag{11}$$

and

$$\mathbf{Q}^{(n)} \equiv \sum_{m\in\mathcal{I}_n} \mathbf{H}^{(n,m)}\mathbf{U}^{(m)}\mathbf{P}^{(m)}\left[\mathbf{U}^{(m)}\right]^{\mathrm{H}}\left[\mathbf{H}^{(n,m)}\right] \tag{12}$$

denotes the total noise and interference and the interference covariance matrices for user n, $\mathbf{P}^{(m)} \equiv \mathrm{diag}\left\{p_1^{(m)}, p_2^{(m)}, \dots, p_{d_m}^{(m)}\right\}, p_d^{(m)} \equiv \mathcal{E}\left|\tilde{x}_d^{(m)}\right|^2 / \Delta_f$, is the normalized transmit power on the d-th stream of user m, $\mathbf{I}_K$is the identity matrix of size K, and $\mathbf{C}_{\star d}$ denotes the d-th column of $\mathbf{C}$. For fair comparison with conventional G.fast systems, we assume that $\tilde{b}_d^{(n)}$ is truncated to $b_{\max}$, if $\tilde{b}_d^{(n)} > b_{\max}$. The bit rate is then calculated by:

$$R^{(n)} = f_s \sum_{d=1}^{d_n} \tilde{b}_d^{(n)} \tag{13}$$

3.2. Third Configuration

In the third configuration, we consider partial signal coordination between users which is the same as the second configuration. However, unlike the second configuration, vectoring is not applied and IA is applied on both the space and frequency dimensions. To be able to apply IA on the space dimension, we need to make some modifications to our earlier formulation. Let $\mathcal{N}_i = \{u_i, u_i + 1, \dots, v_i\}$ denote the set of users in group i, where u_i and $v_i = u_i + N_i - 1$denote the first and last users in group i, respectively. At the transmitter side, the data of all users in group i is arranged in a single vector $\hat{\mathbf{x}}_i \equiv \left[\left(\tilde{\mathbf{x}}^{(u_i)}\right)^{\mathrm{T}}, \dots, \left(\tilde{\mathbf{x}}^{(v_i)}\right)^{\mathrm{T}}\right]^{\mathrm{T}}$, where $\tilde{\mathbf{x}}^{(n)}$ is the n-nth user's data vector of length d_n. The precoder matrix for group i is defined as:

$$\widehat{\mathbf{V}}_{\mathrm{i}} = \begin{bmatrix} \mathbf{V}^{(\mathrm{u_i,u_i})} & \dots & \mathbf{V}^{(\mathrm{u_i,v_i})} \\ \vdots & \ddots & \vdots \\ \mathbf{V}^{(\mathrm{v_i,u_i})} & \dots & \mathbf{V}^{(\mathrm{v_i,v_i})} \end{bmatrix} \tag{14}$$

where $\mathbf{V}^{(m,l)}$ is the $K \times d_l$ sub-matrix for precoding the data vector of user l into the transmitted vector of user m. Note that, $\mathbf{V}^{(m,l)}$ is only defined when users m and l belong to the same group.

Then, the transmitted vector for group i is obtained by

$$\mathbf{x}_{\mathrm{i}} = \widehat{\mathbf{V}}_{\mathrm{i}}\hat{\mathbf{x}}_{\mathrm{i}} \tag{15}$$

which can be written as $\mathbf{x}_i \equiv \left[\left(\mathbf{x}^{(u_i)}\right)^{\mathrm{T}}, \dots, \left(\mathbf{x}^{(v_i)}\right)^{\mathrm{T}}\right]^{\mathrm{T}}$, where $\mathbf{x}^{(m)}$ is the m-th user's transmitted vector of length K. With this notation we have

$$\mathbf{x}^{(\mathrm{m})} = \sum_{\mathrm{l}\in\mathcal{N}_{\mathcal{G}_{\mathrm{m}}}} \mathbf{V}^{(\mathrm{m,l})}\tilde{\mathbf{x}}^{(\mathrm{l})} \tag{16}$$

where $\mathcal{N}_{\mathcal{G}_m}$denotes the set of users in the same group as user m. Therefore, the received signal vector for users n can be written as

$$\begin{aligned}\mathbf{y}^{(n)} &= \sum_{m=1}^{N} \mathbf{H}^{(n,m)}\mathbf{x}^{(m)} + \mathbf{z}^{(n)} \\ &= \sum_{m=1}^{N} \sum_{l\in\mathcal{N}_{\mathcal{G}_m}} \mathbf{H}^{(n,m)}\mathbf{V}^{(m,l)}\tilde{\mathbf{x}}^{(l)} + \mathbf{z}^{(n)}\end{aligned} \tag{17}$$

Noting that $l \in \mathcal{N}_{\mathcal{G}_m} \Leftrightarrow m \in \mathcal{N}_{\mathcal{G}_l}$, we can exchange the order of the first and second sums to obtain

$$\mathbf{y}^{(n)} = \sum_{l=1}^{N} \left[\sum_{m \in \mathcal{N}_{\mathcal{G}_l}} \mathbf{H}^{(n,m)} \mathbf{V}^{(m,l)} \right] \tilde{\mathbf{x}}^{(l)} + \mathbf{z}^{(n)} \\ = \sum_{m \in \mathcal{N}_{\mathcal{G}_n}} \mathbf{H}^{(n,m)} \mathbf{V}^{(m,n)} \tilde{\mathbf{x}}^{(n)}, \qquad (18)$$

where the first sum denotes the desired signal and the second sum denotes the interference. Note that unlike the second configuration, the intra-group crosstalk is included in the sum.

Now consider an arbitrary user m in group $j = \mathcal{G}_m$ and let $\mathcal{N}_j = \{u_j, u_j + 1, \ldots, v_j\}$ denote the set of users in group j. We denote the channel matrix from group j to user n by $\widehat{\mathbf{H}}_j^{(n)} \equiv \left[\mathbf{H}^{(n,u_j)} \colon \mathbf{H}^{(n,u_j+1)} \colon \ldots \colon \mathbf{H}^{(n,v_j)}\right]$ and the extended precoder matrix for user nn by $\widehat{\mathbf{V}}^{(n)} \equiv \left[\left(\mathbf{V}^{(u_j,n)}\right)^{\mathrm{T}} \vdots \left(\mathbf{V}^{(u_j+1,n)}\right)^{\mathrm{T}} \vdots \cdots \vdots \left(\mathbf{V}^{(v_j,n)}\right)^{\mathrm{T}}\right]^{\mathrm{T}}$. To perfectly align all the interference to user n, the following conditions must hold:

$$\mathrm{span}\left(\widehat{\mathbf{H}}_{\mathcal{G}_\mathrm{m}}^{(\mathrm{n})} \widehat{\mathbf{V}}^{(\mathrm{m})}\right) = \mathrm{span}\left(\widehat{\mathbf{H}}_{\mathcal{G}_\mathrm{l}}^{(\mathrm{n})} \widehat{\mathbf{V}}^{(\mathrm{l})}\right), \forall \mathrm{l}, \mathrm{m} \neq \mathrm{n} \qquad (19)$$

where l and m denote two interfering users to user nn which may or may not belong to the same group as user n 's group.

At the receiver side, a $K \times d_n$ decoder matrix $\widehat{\mathbf{U}}^{(n)}$ is used by user $n, n = 1, \ldots, N$, to suppress the interference:

$$\tilde{\mathbf{y}}^{(\mathrm{n})} = \left[\widehat{\mathbf{U}}^{(\mathrm{n})}\right]^{\mathrm{H}} \mathbf{y}^{(\mathrm{n})} \qquad (20)$$

Note that since the users are not (partially) coordinated at the receiver side, $\widehat{\mathbf{U}}^{(n)}$ is applied on the frequency dimension only. Finally, $\tilde{\mathbf{y}}^{(n)}$ is used to recover the data vector for user n.

The bit rate of user nn is obtained by equations (9) to (13) by replacing $\mathbf{V}^{(n)}, \mathbf{U}^{(n)}$, and $\mathbf{H}^{(n,m)}$ by $\widehat{\mathbf{V}}^{(n)}, \widehat{\mathbf{U}}^{(n)}$, and $\widehat{\mathbf{H}}_{\mathcal{G}_m}^{(n)}$, respectively, where the set of interfering users for user n is $\mathcal{I}_n = \{1, \ldots, n-1, n+1, \ldots, N\}$.

2.3. Joint Configurations 2 and 3

In configuration 3, IA is applied in the space and frequency domains jointly. Configuration 1, in which IA is only applied in the frequency domain, is a special case of configuration 3 where the lack of coordination between users can be incorporated by assuming that the coordination groups consist of single users, i.e., $G = N$ and $\mathcal{N}_i = \{i\}$ for $i = 1, \ldots, N$. Likewise if we apply configuration 3 on the tones separately, we obtain a sub-configuration of configuration 3 where IA is applied on the space dimension only. In this case, we obtain G precoder matrices of size $N_i \times N_i$ for $i = 1, \ldots, G$. This arrangement called "sub-configuration 3" provides a solution to the joint precoder design problem in DSL IF-BC channels [5] and can be implemented using vectoring. Sub-configuration 3 has the same computational complexity as PV but as we will see in Sec. 4, it outperforms vectoring based on the DP considerably.

Moreover, by replacing the per-tone diagonalizing precoders exploited in configuration 2 with the precoders obtained by sub-configuration 3, we obtain a new configuration, namely, configuration 2 and sub-configuration 3 (JC2&SC3). Similar to configuration 2, after applying per-tone precoders, IA is applied in the frequency domain on the modified channel sub-configuration 2 ($\Delta K = 1$) that has the same computational complexity as configuration 2 but it almost achieves the performance of configuration 3 based on our simulation results in Sec. 4.

2.4. Calculating Precoders and Decoder Matrices

When each user is equipped with multiple antennas (or tones in the DSL context), closed-form solutions for interference alignment (IA) remain elusive for networks with more than three users [8]. However, several algorithms exist that achieve approximate IA solutions through iterative techniques. In [11], two distributed algorithms, minIL and maxSINR, are proposed to determine the precoder and decoder matrices that align interference in networks with an arbitrary number of nodes and multiple antennas at each node. In [12], the maxSum-Rate algorithm has been proposed to achieve maximum achievable sum rate as well as zero interference leakage. These algorithms, which are explained shortly, can be exploited directly to calculate the precoder and decoder matrices for the first and second configurations. However, for the third configuration, we need to use algorithms specifically designed for IF-BC channels.

So far, several algorithms have been proposed for IA in IFBC channels. In [15], an algorithm has been proposed which achieves optimal DoF. However, it can be applied to partially coordinated DSL systems where users are divided into at most two groups with a maximum of two users in each group. A sub-optimal but efficient technique has been proposed in [16]. Another algorithm has been proposed in [17] for systems with two groups with arbitrary number of users per group. The algorithms proposed in [18] and [19] are extensions of the algorithm in [15] to systems with multiple groups and to systems with multiple groups and multiple users per group, respectively. Unfortunately, when applied to DSL systems, these techniques reduce the number of data streams per-user substantially.

In this paper, we examine three iterative IA algorithm for crosstalk mitigation in the first and the second configurations. For the third configuration, we use the scheme proposed in [15] for the IF-BC channel which is not discussed here for brevity.

3. Simulation Results

In this section, we present simulation results comparing the performance of IA in the proposed configurations with PV. We also propose two sub-configurations to improve the performance and reduce the computational complexity of the system based on the third configuration. In our simulations, we use the 26 AWG cable model and the G.fast crosstalk model [20] to calculate the DSL channel matrix. The frequency tone spacing is set to $\Delta f = 51.75\mathrm{kHz}$ and the DMT symbol error rate is set to $f_s = 48\mathrm{kHz}$. We use G.fast 106a bandplan [21]. The SNR gap is set to 10.75 dB. We use a flat $-76\mathrm{dBm/Hz}$ PSD in our simulations and the PSD of noise is set to $-140\mathrm{dBm/Hz}$. If not mentioned otherwise, IA is applied on $L = 200$ subsets where the subset size is $\Delta K = 10$, the DoF in each subset is $\delta = d_n/L = \Delta K = 10, \forall n$, and the minIL algorithm is used to calculate the precoder and decoder matrices. We consider four scenarios in this paper:

Scenario 1: This scenario consists of 6 users in three groups of size 2, where $\mathcal{N}_1 = \{1,2\}, \mathcal{N}_2 = \{3,4\}$ and $\mathcal{N}_3 = \{5,6\}$, as illustrated in Fig. 1. The loop lengths of the users are 190, 200, 150, 160, 90, and 100 m respectively. Groups two and three are offset by 40 and 70 m from the CO, respectively. Considering the loop lengths and relative distance of interfering users to the victim users, the users in groups one to three are labeled low, medium, and high SNR users, respectively.

Using this scenario, in Fig. 2 we have evaluated the performance of the IA algorithms introduced in Sec. 3.4 for the first configuration. The achievable bit rate of users is plotted vs. loop length for the three IA algorithms and the case without using IA (labeled "No IA"). Using IA and depending on the loop length, the achievable bit rates of the users have increased by 154% to 511%. Note that these rates are achieved using the first configuration, that is vectoring is not applied and (partial) signal level coordination is not required. As can be seen, the maxSINR algorithm achieves much higher rates than the minIL and maxSum-Rate algorithms for medium- and low-SNR users. However, the minIL algorithm has the least complexity in computing the precoder and decoder matrices.

The effect of ΔK on the sum achievable rates of the users in Scenario 1 is plotted in Fig. 3. By setting $\Delta K = 6$ and 10 about 95 and more that 97 of sum achievable rate is achieved.

Scenario 2: In this scenario, we consider 10 co-located G.fast lines with equal loop lengths. We have simulated this scenario at the eight different loop lengths $0,70,\dots,190$ m. Simulation results for Scenario 2 is illustrated in Fig. 4 where the achievable bit rates in percentage of those achieved in a crosstalk- free network[i] are plotted vs. loop length**.** IA using configuration 1 is compared to the case where no crosstalk cancellation scheme is applied and the case where FV is applied.

As can be seen, IA achieves from 55% to 77% of the crosstalkfree network rate (CFNR) without any need for signal level coordination among users. This is from 2.0 to 2.7 times higher than the rates achieved when no crosstalk cancellation scheme is exploited. Moreover, since $\Delta K = N = 10$, the total computational complexity of IA is two times as vectoring in this scenario. But unlike vectoring, IA can be applied by users in a distributed fashion, which reduces the computational complexity burden per each processing unit by a factor of 20.

Scenario 3: This scenario consists of 30 users in three groups of size 10 (i.e., $\mathcal{N}_1 = \{1,2,\dots,10\}, \mathcal{N}_2 = \{11,\dots,20\}$, and $\mathcal{N}_3 = \{21,\dots,30\}$) as follows: The loop lengths of the users in the groups are $50, 60, 70, \dots, 140$ m. The second and third groups are offset by 35 m and 70 m from the CO, respectively. A schematic of this scenario is plotted in Fig. 5. The first and the second configurations are applied in this scenario.

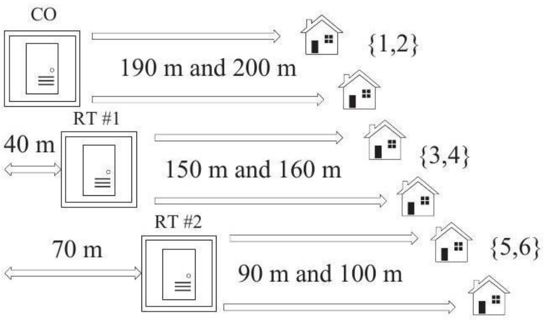

Fig. 1. A schematic of Scenario 1

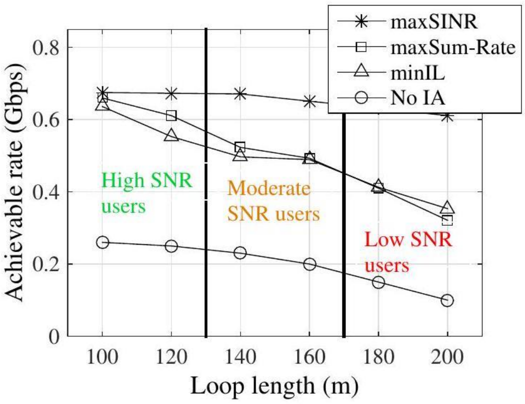

Fig. 2. The achievable bit rates vs. loop length for the users in Scenario 1 using minIL, maxSINR and maxSum-Rate algorithms compared to the "No IA" case

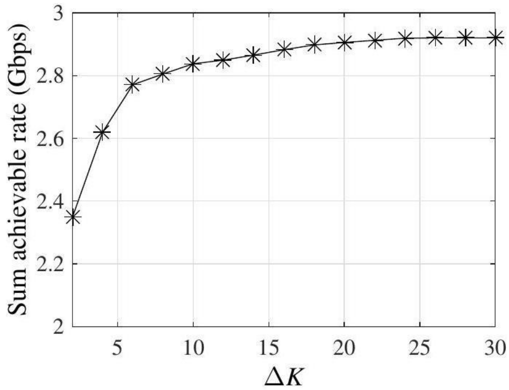

Fig. 3. The sum achievable rates vs. ΔK in Scenario 1.

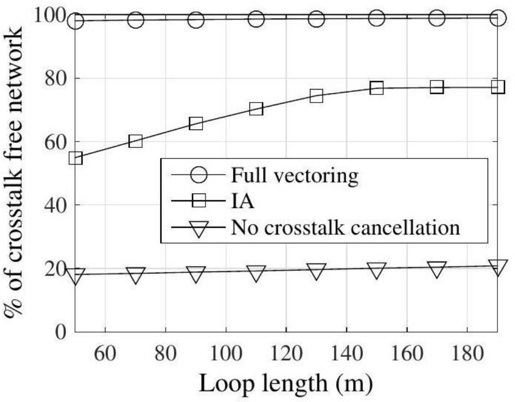

Fig. 4. The percentage of crosstalk free network vs. loop length for Scenario 2 using IA compared to Full vectoring and no crosstalk cancellation scheme.

Simulation results for Scenario 3 is summarized in Table 1. The sum achievable rate of users is reported for the following cases: 1) full vectoring, 2) partial vectoring (without IA), 3) IA using Configuration 1, 4) IA using Configuration 2, and 5) no crosstalk cancellation scheme. The numbers are also reported in percentage of the sum achievable rates in a crosstalk-free network. Full vectoring can achieve 98.03% of the crosstalk-free network rate (CFNR). Using configuration 1, IA can independently achieve 41.83% of the CFNR which is 33.86% higher than the achievable rates when no crosstalk cancellation scheme is used. When PV and IA are applied jointly using configuration 2, the sum achievable rate increases to 77.46% of the CFNR. Finally, when PV is applied without IA, the sum achievable rates are only 23.57% of the CFNR.

Scenario 4: This scenario consists of 4 users in the two groups $\mathcal{N}_1 = \{1,2\}$ and $\mathcal{N}_2 = \{3,4\}$ as shown in Fig. 6. The loop lengths of the users are $60, 70, 140,$ and 150 m, respectively. Group two is offset by 40 m. Simulation result for various schemes is summarized in Table 2. These include the sum achievable rates for PV using the DP as well as IA in configurations 1 to 3, and the results for the two sub-configurations proposed in Sec. 3.3. When no crosstalk cancellation scheme is exploited, we achieve $12.25\%$of the CFNR. Using PV without IA, we achieve 36.30% of the CFNR. When IA is applied using configuration 1, we achieve 38.08% of the CFNR. Using configuration 2 in standard form, where we use the DP for PV, the achievable rates increase considerably to 69.91%. Still, configuration 3 outperforms configuration 2 noticeably by reaching 87.95% of the CFNR.

Using sub-configuration 3 where $\Delta K = 1$ and IA is merely applied on the space dimension, we achieve 51.23% of CFNR. This is smaller than the achievable rates by configurations 2 and 3; however, considerably higher than the rates achieved by using the DP. Note that sub-configuration 3 obtained by IA with $\Delta K = 1$ is a vectoring scheme and therefore, has the same computational and implementation complexity as PV using DP. Per-tone bitloadings of user 1 is plotted vs. tone index in Fig. 7. As can be seen, DP outperforms sub-configuration 3 at lower frequencies. This is because at low frequencies inter-group, crosstalk is negligible compared to the background noise and $\beta \approx 1$ due to the row-wise diagonal dominance (RWDD) property of the downstream DSL channel. Therefore, pushing inter-group crosstalk into zero forcefully using IA does not necessarily result in higher rates. However, by increasing the frequency, the inter-group crosstalk power increases while the RWDD property vanishes. As can be seen, the bit loadings obtained by sub-configuration 3 are considerably larger than those obtained by the DP at high frequencies. The achievable rate by sub-configuration 2, where we use sub-configuration 3 as for PV instead of the DP, is 86.71% which is very close to that of Configuration 3, i.e., 87.45%. This makes this scheme an attractive option as its implementation complexity is the same as configuration 2 and considerably smaller than configuration 3.

Finally, we have studied the effect of number of data streams to ΔK on the achievable data rate in Fig. 8 for Scenario 4 in configuration 3. As can be realized, although perfect IA is achieved when the number of streams in each subset is $\Delta K/2$, maximum bit rate is achieved when the number of streams is set equal to ΔK.

Table (1): Sum achievable rates by various crosstalk cancellation schemes for scenario 3

Scheme		Sum rate (Gbps)	% of xtalk free
Vectoring	Full	31.68	98.03
	Partial	7.619	23.57
IA	Config. 1	13.522	41.83
	Config. 2	25.03	77.46
No crosstalk cancel. Scheme		2.57	7.97

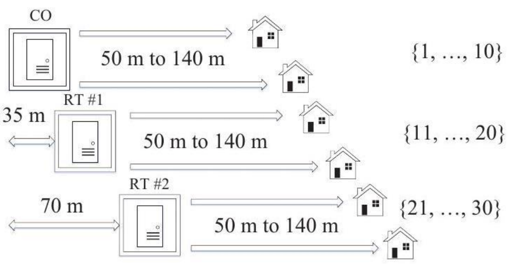

Fig. 5. A schematic of Scenario 3.

Table (2): Sum achievable rates by the proposed schemes for scenario 4

Scheme		Sum rate (Gbps)	% of xtalk free
Partial vectoring (DP)		1.413	36.30
IA	Config. 1	1.511	38.08
	Config. 2 (DP for PV)	2.779	69.91
	Config. 3	3.431	87.45
	Sub-Config. 3 (ΔK = 1)	1.953	51.23
	Sub-Config. 2 (ΔK = 1 for PV)	3.381	86.71
No crosstalk cancel. Scheme		0.446	12.25

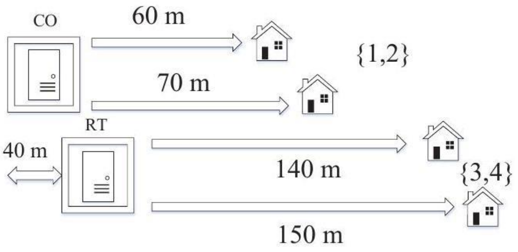

Fig. 6. A schematic of Scenario 4

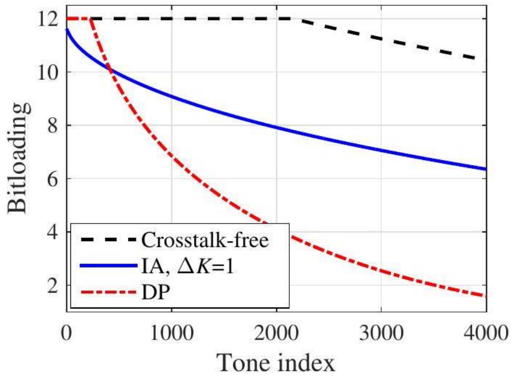

Fig. 7. The achievable bit rate of user 1 vs. frequency tones in Scenario 4 using Sub-Configuration $3(\Delta K = 1)3(\Delta K = 1)$ compared to PV using the DP

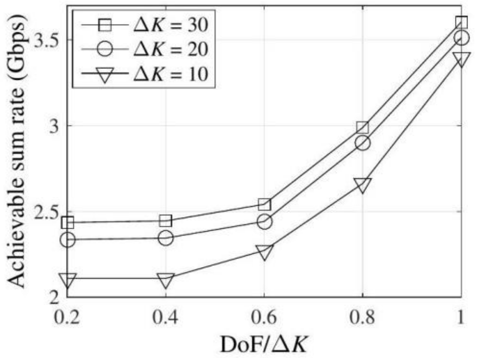

Fig. 8. The sum achievable rate vs. $\mathrm{DoF}/\Delta K\mathrm{DoF}/\Delta K$ for Scenario 4 using configuration 3

4. Conclusions and Future Work

In this paper, we proposed several configurations for crosstalk cancellation in DS G.fast networks using IA. Moreover, we studied the performance of three IA algorithms, namely the minIL, the maxSINR, and the maxSum-Rate algorithms in G.fast networks. Our simulation results show that the maxSINR algorithm achieves higher bit rates than the other algorithms for low and medium SNR loops. However, the minIL algorithm was chosen in our simulations as it has the least computational complexity. We studied the performance of IA in four different G.fast scenarios. Configuration 1 does not require signal-level coordination among users and IA is implemented in a distributed fashion. Using IA in configuration 1, 38 to 77% of CFNR was achieved. Configurations 2 and 3 are applicable when only partial signal-level coordination is available between users. Largest bit-rates were achieved when configuration 3, in which IA is applied jointly on space and frequency dimensions, was used. Two new sub-configurations were proposed in this paper as well. By applying IA only on the space dimension, we obtained a solution for join precoder design problem in partially coordinated DS DSL networks, namely, sub-configuration $3(\Delta K = 1)$. The proposed solution outperforms the DP considerably. Then by replacing the diagonalizing precoders in configuration 2 with precoders obtained by sub-configuration $3(\Delta K = 1)$, we obtained sub-configuration $2(\Delta K = 1)$. Sub-Configuration 2 $(\Delta K = 1)$ has the same computational complexity as configuration 2. Our simulation results show that sub-configuration 2 ($\Delta K = 1$ for PV) outperforms configuration 2 substantially and nearly achieves the bit-rates achieved by configuration 3.

Our simulation results show that the maximum bit rates are achieved when the number of streams per each user is the same as the number of frequency tones. In configuration 3, we use IA algorithms designed for the IF-BC channel. Unfortunately, when used in DSL, most of these algorithms result in considerable reduction in the number of streams per user. Therefore, an important topic to study is the development of IA algorithms for DSL IF-BC channels, which do not result in the reduction of the number of streams. Another topic for investigation could be the use of optimal joint precoders as for PV in configuration 2. Following the results obtained by sub-configuration $2(\Delta K=1)$ in which we use IA to find the precoders, it is expected that using optimal precoders would increase the achievable rates even more.

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[i] We use the term "crosstalk-free network" to denote a network with zero crosstalk couplings but the same direct channel gains as the network being considered.