International Economic Studies
Vol. 51, No. 1, 2021
pp. 15-24
Received: 15-12-2020   Accepted: 09-06-2021

Is Friedman Still Right?

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Abstract
This paper has investigated the role of search and matching frictions in a monetary model and examined if the Friedman rule, advocating a rate of deflation equal to real interest rate, is still right. We defined a dynamic programming problem in which money is situated in the model by cash in advance constraint, and used a numerical method (value function iteration method) to solve the pre-mentioned problem. Also, in this paper, the concept of the homogenous agent is substituted by the heterogeneous agent, and there are two groups of agents, namely unemployed and employed agents. The difference between the two divergence groups is indicated by different constraints in this study. According to our model, the Friedman rule will not maximize the aggregate welfare of the assumed society with this new friction. It is noteworthy that the parameters of the numerical model have been derived from the United States economy and the essay is theoretical. The results can be applied in different economies with their specific parameters. Also, the study offers some implications to central banks.

Keywords: Monetary Policy, Unemployment, Search and Matching Theory.
JEL Classification: E52, E24, C78.

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1- Introduction:

The search and matching theory has entered economic literature since the 1970s. This theory leads to develop different fields of economics whose main effect and development are associated with the labor economy. The search and matching theory tries to answer some important questions in the labor market, such as why unemployment exists and what should we do about it. Some unemployed workers are searching for jobs; moreover, there are some firms with empty vacancies which are searching for workers. When suitable workers and vacancies meet each other, the matching could occur. Because of this process, the labor markets usually are not clear and there is unemployment in the labor market.

In this paper, we aim to reevaluate the Friedman rule with the search and matching frictions and answer an absolutely theoretic problem. Friedman (1969) introduced a new rule for monetary policy. He expressed that the opportunity cost of holding money for an agent (interest rate) should be equal to the marginal cost of producing money (which is zero), so the nominal interest rate should be zero under these circumstances. Let the nominal interest rate be zero then according to the Fisher equation, a rate of deflation (real interest rate) will exist.

Under Friedman's rule, the money growth rate is equal to the household discount factor minus one. This growth rate would be negative, for the discount factor is less than one. Freidman showed that such a rule will maximize social welfare. Here, we form a model and show that Friedman's rule will not maximize the aggregate welfare when we have search and matching frictions. For the intuition of this issue, we should refer to our model. We can recognize three channels for the effect of the money growth rate on social welfare. First, in this paper, we use the concept of the heterogeneous agent and introduce two types of agents (because of search and matching frictions), namely employed and unemployed groups. Employed people hold more real money balances than the unemployed and if the money growth rate increases, the more money balances one holds, the more contribution of inflation tax will be made. In other words, when the government gives the same lump-sum transfer to both of the agents and the money growth rate increases, there is a redistribution of income from employed to unemployed. This channel can bring a Pareto optimal for the society.

Second, the money growth rate has a negative effect on social welfare. Such effect on the equations of agents in the rest of the present paper is obvious

which is because of the opportunity cost of holding money. It is noteworthy that the inflation rate is the opportunity cost of holding money acting as a cost for money balances. Third, the money growth rate has a positive effect on the consumption of agents through lump-sum transfers of government.

There are lots of related studies in this field. Cheron and Langot (2000) combine search and matching friction and nominal rigidities. They show how this framework can produce the Philips Curve and the Beveridge Curve. This study would be the first which uses search and matching frictions in the economic models. In most related literature, there are two main branches of studies. One focuses on the monetary policy with search and matching and other frictions and the other concentrates on the test of optimality of the Friedman rule under different situations. In the former branch of these studies, we can refer to Walsh (2005), Sala, Soderstrom, and Trigari (2008), Gertler, Sala, and Trigari (2008), Faia (2008), Blanchard and Gali (2010), and Ravenna and Walsh (2011). In the latter part of the studies, Chari, Christiano, and Kehoe (1991), Khan, King, and Wolman (2003), Grohe and Uribe (2004), Shaw, Chang, and Lai

(2006), and Gahvari (2012) are some of the important ones.

In the next sections of this essay model, the results, and conclusion of our model will be considered.

2- Model:

In our model, we will consider two types of agents. One of them is employed people and the other is unemployed. These two types of agents because of their job status have different behaviors that we should notice these differences in our modeling.

In the first step, we should have a road map through the timing of the model:

1- At the beginning of the period, two types of agents are employed or unemployed. Unemployed people may find a job during the period by an exogenous probability (π) and will be considered employed in the next period. Also, employed ones may lose their job by an exogenous probability (ρ) and will be unemployed in the next period.

2- The agents enter with m_t amount of money and receive τ_t (lump sum transfer) from the government and buy goods with this money at the beginning of the period (cash in advance constraint).

3- Unemployed will not spend all of m_t andτ_t, but the CIA is binding for employed.

4- Employed will receive $w_t n_t^s$, during the period.

5- All the agents decide for m_{t+1}.

Now we should form the model under this timing.

2-1- Employed People

We can write the problem of employed people through defining value function as follows:

$$J(1,m_t) = \max \quad u(c_t) + f(l_t) + \beta(1-\rho)J(1,m_{t+1}) + \beta(\rho)J(0,m_{t+1}) \qquad (1)$$

$J(1,m_t)$, is the value function of employed people. The state variable of this equation is m_t (the money of the current period), and the control variable is m_{t+1}(the money which the agent decides to hold for the next period).

In this problem, l_t, is the leisure of agent, and ρ, is the exogenous rate of job destruction. The agent should maximize this problem subject to:

$$p_t c_t + M_{t+1} = M_t + W_t n_t^s + p_t \tau_t \qquad \text{Budget constraint} \qquad (2)$$

$$p_t c_t = p_t \tau_t + M_t \qquad \text{CIA[1] constraint} \qquad (3)$$

- W_t, is the wage of employed.
- n_t^s, is their optimal labor supply.
- $p_t \tau_t$, is the lump sum transfer of the government.
- Cash in Advance constraint is binding for employed people, because they have another source of income (their wage) during the period.

These two constraints are in nominal terms and we should write them in real terms:

$$c_t + m_{t+1}(1+\mu) = m_t + w_t n_t^s + \mu m_t^s \qquad \text{Budget constraint} \qquad (4)$$

$$c_t = \mu m_t^s + m_t \qquad \text{CIA constraint} \qquad (5)$$

There are some new terms in the above equations. For elaborating them we should introduce the following equations:

$$\begin{cases} p_t \tau_t = M_{t+1}^s - M_t^s \\ M_{t+1}^s - M_t^s = \mu M_t^s \end{cases} \rightarrow p_t \tau_t = \mu M_t^s \qquad (6)$$

$p_t \tau_t = M_{t+1}^s - M_t^s$, is the government constraint and $M_{t+1}^s - M_t^s = \mu M_t^s$, is the equation of money growth.

Equation (5) is the consumption of employed people and we can find an important relation from this equation:

$$m_{t+1} = \frac{w_t n_t^s}{(1+\mu)} \qquad (7)$$

μ, is the inflation rate of the economy which has a positive amount generally. This equation shows the opportunity cost of holding money.

m_{t+1}, is the optimal money which

[1]- Cash in Advance

employed people decide to transfer to the next period.

Now, we know that the basic employed problem for solving is as follows:

$$J(1,m_t) = \max \quad u(c_t) + f(l_t) + \beta(1-\rho)J(1,m_{t+1}) + \beta(\rho)J(0,m_{t+1})$$

$$s.t.\begin{cases} c_t + m_{t+1}(1+\mu) = m_t + w_t n_t^s + \mu m_t^s \\ c_t = \mu m_t^s + m_t \end{cases}$$

In other words:

$$(1,m_t) = \max \quad u(\mu m_t^s + m_t) + f\left(1 - (\frac{m_{t+1}(1+\mu)}{w_t})\right) + \beta(1-\rho)J(1,m_{t+1}) + \beta(\rho)J(0,m_{t+1})$$

2-2- Unemployed People

The above problem for the unemployed has some changes:

$$J(0,m_t) = max \quad u(c_t) + f(l_t) + \beta\,\pi\,J(1,m_{t+1}) + \beta(1-\pi)J(0,m_{t+1}) \qquad (8)$$

$J(0,m_t)$ is the value function of unemployed people. π is the exogenous rate of job finding. In this part $l_t = 1$ (if we normalize the whole hours of a day to 1 and remember that $l_t = 1 - n_t$, because of the unemployment of agents, $n_t = 0$ so $l_t = 1$)

$p_t c_t + M_{t+1} = M_t + p_t \tau_t$	Budget constraint	(9)
$p_t c_t < p_t \tau_t + M_t$	CIA constraint	(10)

The first difference of this part with the employed people is the absence of wage term in the budget constraint which is obvious because of unemployment. The next difference is associated with the CIA constraint which is not binding. The above equations are in nominal terms and we change them to real terms in the following equations.

$c_t + m_{t+1}(1+\mu) = m_t + \mu m_t^s$	Budget constraint	(11)
$c_t < \mu m_t^s + m_t$	CIA constraint	(12)

The consumption of unemployed people will be extracted from 11:

$$c_t = \mu m_t^s + m_t - (1+\mu)m_{t+1} \qquad (13)$$

And as we explained for the other agent, for unemployed people we can write:

$$J(0,m_t) = \max \quad u(\mu m_t^s + m_t - (1+\mu)m_{t+1}) + f(l_t) + \beta\pi\,J(1,m_{t+1}) + \beta(1-\pi)J(0,m_{t+1})$$

The explicit form of utility and leisure functions are as follows, which both of them are strictly increasing and concave,

$$u(c_t) = \frac{{c_t}^{1-\sigma}}{1-\sigma}\ ,\ f(l_t) = \frac{(1-n_t)^{1-\eta}}{1-\eta}, (l_t = 1 - n_t)$$

The employment in the next period can be written according to Equation (14).

$$N_{t+1} = N_t(1-\rho) + (1-N_t)\,\pi \qquad (14)$$

In equation (14), N_t is the number of employed workers in the current period and $(1-N_t)$ is the normalized value of unemployed workers. On one side, employed workers may lose their job by an exogenous probability (ρ) and consequently will be unemployed in the next period. Hence, $(N_t(1-\rho))$ is the portion of current period workers who saved their job. On the other side, unemployed people may find a job during the period by an exogenous probability (π) and will be considered employed in the next period, so $((1-N_t)\,\pi)$, is the portion of unemployed workers finding a job.

We can compute the employment of the society in the steady-state as follows:

$$N = N(1-\rho) + (1-N)\pi \rightarrow N = \frac{\pi}{\rho+\pi} \qquad (15)$$

Now, we should calculate the aggregate money and aggregate welfare of the society. Each of these variables consists of two parts. One is about the money and welfare of employed and the other is associated with unemployed people.

$$\begin{aligned} aggregate\ money &= N * \left(\frac{wn}{1+\mu}\right) + N\rho * P\_0m\left(\frac{wn}{1+\mu}\right) \\ &+ N\rho(1-\pi) * P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right) \\ &+ N\rho(1-\pi)^2 \\ &* P\_0m\left(P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)\right) \\ &+ N\rho(1-\pi)^3 \\ &* P\_0m\left(P\_0m\left(P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)\right)\right) \\ &+ N\rho(1-\pi)^4 \\ &* P\_0m\left(P\_0m\left(P\_0m\left(P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)\right)\right)\right) \\ &+ \cdots \end{aligned} \tag{16}$$

$$aggregate\ money\_E = N * \left(\frac{wn}{1+\mu}\right) \tag{17}$$

$$\begin{aligned} aggregate\ money\_U &= N\rho * P\_0m\left(\frac{wn}{1+\mu}\right) + N\rho(1-\pi) \\ &* P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right) + N\rho(1-\pi)^2 \\ &* P\_0m\left(P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)\right) \\ &+ N\rho(1-\pi)^3 \\ &* P\_0m\left(P\_0m\left(P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)\right)\right) \\ &+ N\rho(1-\pi)^4 \\ &* P\_0m\left(P\_0m\left(P\_0m\left(P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)\right)\right)\right) \\ &+ \cdots \end{aligned} \tag{18}$$

Two distinct parts of aggregate money which are related to employed aggregate money and unemployed aggregate money defined in Eq. (17) and Eq. (18), respectively. For the aggregate money of employed people, we need to multiply N (the number of employed people in the steady state) in $\frac{wn}{1+\mu}$ (the optimal money for transferring to the next period). In other words, $N\left(\frac{wn}{1+\mu}\right)$ is the aggregate money held by employed people which is elaborated in Eq. 17. For calculating 'the unemployed aggregate money', we should define a policy function (P_0m). The policy function will show the next period's value for every amount of the previous period. For instance, $(P\_0m\left(\frac{wn}{1+\mu}\right))$ is the value of the function at the point $(\frac{wn}{1+\mu})$. If we look at the society at a time point, there are different groups of unemployed workers according to the number of unemployment periods.

$(N\rho * P\_0m\left(\frac{wn}{1+\mu}\right))$ stands for the aggregate amount of money that workers experiencing one period of unemployment. $N\rho(1-\pi) * P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)$ measures the amount of money which is held by unemployed workers experiencing two periods of unemployment, and so on.

In the next lines, aggregate welfare is defined. Although the concept of extracting this equation is similar to the aggregate money equation, the policy functions are a bit different.

$$\begin{aligned} aggregate\ welfare &= N * J\_1m\left(\frac{wn}{1+\mu}\right) + N\rho * J\_0m\left(\frac{wn}{1+\mu}\right) \\ &+ N\rho(1-\pi) * J\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right) \\ &+ N\rho(1-\pi)^2 \\ &* J\_0m\left(P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)\right) \\ &+ N\rho(1-\pi)^3 \\ &* J\_0m\left(P\_0m\left(P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)\right)\right) \\ &+ N\rho(1-\pi)^4 \\ &* J\_0m\left(P\_0m\left(P\_0m\left(P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)\right)\right)\right) \\ &+ \cdots \end{aligned} \tag{19}$$

The aggregate welfare of the society has two different functions. J_1m, is the value function of employed people, and J_0m, is the value function of unemployed.

$$aggregate\ welfare\_E = N * J\_1m\left(\frac{wn}{1+\mu}\right) \tag{20}$$

$$aggregate\ welfare\_U = N\rho * J\_0m\left(\frac{wn}{1+\mu}\right) + N\rho(1-\pi) * J\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right) + N\rho(1-\pi)^2 * J\_0m\left(P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)\right) + N\rho(1-\pi)^3 * J\_0m\left(P\_0m\left(P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)\right)\right) + N\rho(1-\pi)^4 * J\_0m\left(P\_0m\left(P\_0m\left(P\_0m\left(P\_0m\left(\frac{wn}{1+\mu}\right)\right)\right)\right)\right) + \cdots \quad (21)$$

2-3- The numerical Method

Value function iteration is among the most prominent methods to solve Dynamic General Equilibrium (DGE) models (Heer & Maubner, 2008). To our best knowledge, this method is introduced by Taylor and Uhlig (1990). There is an important theorem in dynamic programming. This theory states that if an operator has the contraction property, then for every initial condition a unique fixed point will exist. We can conclude from the contraction mapping theorem if the value function satisfies the contraction property, there is a unique fixed point. The conditions which make a value function to have contraction property are stated by Blackwell's sufficiency conditions.

As soon as we understand that the fixed point exists and is unique, we can use a numerical method to solve the Bellman equation. We use the method of value function iteration for solving the bellman. This method has an algorithm with some steps as follows:

1- We should make a grid of feasible values for the state variable (m)

$m = \{m_1, \dots, m_n\}$

2- Now we make an initial guess for value function, $V^0(m)$, this guess is a vector that has a value for every possible value of the state variable.

3- In this step, we can calculate $V^1(m)$ through initial the guess of the value function and the grid of state variable.

4- Now we should compare $V^0(m)$ and $V^1(m)$, if they are not close to each other we should take $V^1(m)$ as an initial conjecture and go back to the second step.

5- We should continue this algorithm until

$\lim_{n\to\infty} \|V^n - V^{n-1}\| = 0$

As it was expressed, for solving the problem which was defined, we used value function iteration. This method will only consider the discrete values in the grid of the state variable while the true maximum may be between these values. For solving this problem, we used the interpolation technique in our codes.

2- 4 - Parameters Values

We consider quarterly values for the parameters which are used in the model. The following table shows the values of the parameters.

Table1. Parameters Values

Matching parameters		Household parameters		
$\rho = 0.12*$	$\pi = 0.7*$	$\sigma = 0.7$	$\beta = 0.99*$	$\eta = 2$

*These values are driven from Blanchard and Gali (2010)

Source: Authors

β, is a parameter describing an agent's preferences and its amount in most studies is 0.99, as reported in Blanchard and Gali' study (2010). Furthermore, job finding rate (π) and inflow to the unemployment rate (ρ) are calibrated by Blanchard and Gali (2010). The explicit form of the utility function and labor supply of households in this study is called Constant Relative Risk Aversion (CRRA). The parameters of these functions (σ *and* η) measure the degree of relative risk aversion and have a value greater than zero (Walsh, 2010).

3- Results

In this section, we will present the main results of the model.

As it is illustrated in Figure (1), the maximum amount of aggregate welfare occurs in a small positive value of μ, which is opposed to Friedman's rule. We have introduced three channels for the effect of money growth rate on social welfare in the introduction part. In our model, the new money growth rate which is welfare-maximizing has a small positive value for the labor market, and different agents have been entered into the model which can improve the related channels and highlight the positive effect of money growth rate on the social welfare.

In the next figures, we let this small value equal to zero.

In Figure 2, we compare the value functions of employed people in the optimum μ of our model and the optimum μ which was proposed by Friedman. As this figure shows the value function in the optimum of our model has a higher level. Figure 3 compares the optimum μ of the model with a higher μ (to show that the suggested aggregate welfare function is not an increasing function to μ) and shows that again, in the optimum of the model, the value function has a higher level.

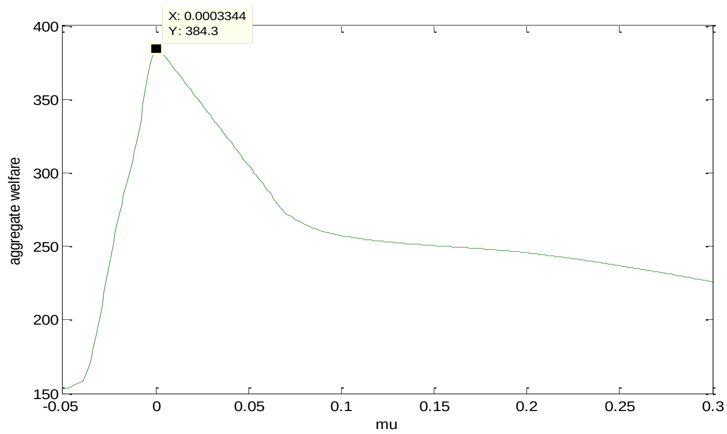

Figure1: the Aggregate Welfare

Source: Authors

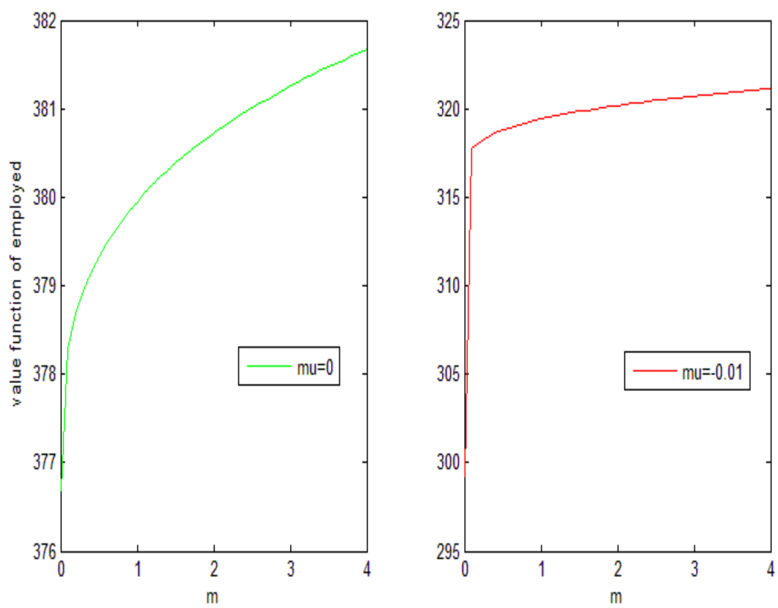

Figure 2: Different Levels of Employed Value Functions

Source: Authors

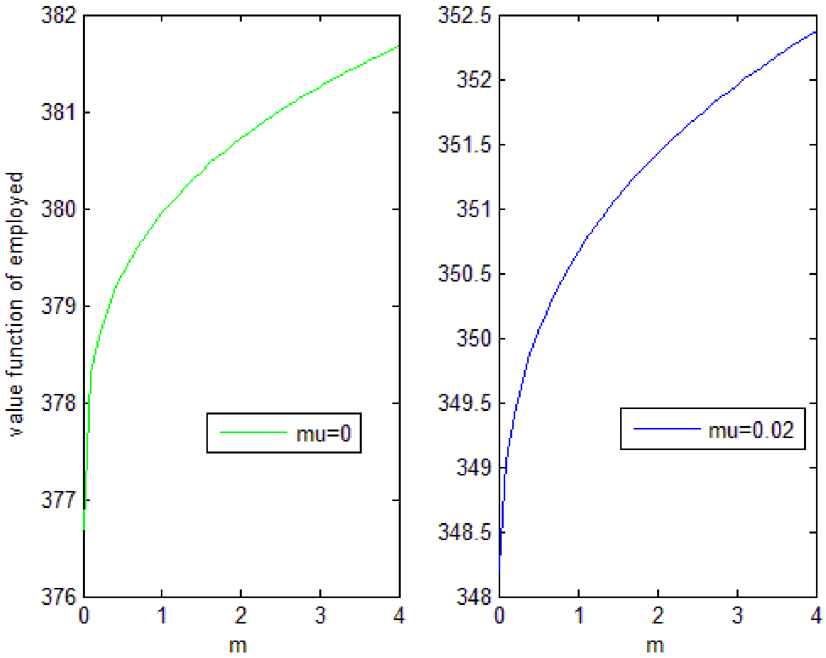

Figure 3: Different Levels of Employed Value Functions
Source: Authors

4- Conclusion

In this paper, we re-evaluate the Friedman rule and add new friction to this rule, also try to answer a theoretical problem. Under this new friction, some new aspects came to our model and the results show that the Friedman rule is not welfare maximizing anymore. We can recognize three channels for the effect of the money growth rate on social welfare. First, in this paper, we use the concept of the heterogeneous agent and introduce two types of agents (because of search and matching frictions), namely employed and unemployed agents. Employed people hold more real money balances than unemployed and if the money growth rate increases. The more money balances one holds, the more contribution of inflation tax will be made. In other words, when the government gives the same lump-sum transfer to both of the agents and the money growth rate increases as if there is a redistribution of income from employed to unemployed. This channel can bring a Pareto optimal for the society. Second, the money growth rate has a negative effect on welfare. This effect on the equations of agents in the paper is obvious which is because of the opportunity cost of holding money. Third, the money growth rate has a positive effect on the consumption of agents through lump-sum transfers of government. The net effect of the money growth rate on social welfare depends on the sum of these effects. In our model, the new money growth rate which is welfare-maximizing has a small positive value which is opposed to the Friedman rule. This change is because of entering the labor market and introducing different agents in the model which can improve the first channel. The results can be applied in different economies with their specific parameters and offered some advice to central banks.

We did this study with exogenous parameters and it can be done by endogenous parameters in the next studies which will present new information.

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