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## TRANSITIVE $t$ -DESIGNS AND STRONGLY REGULAR GRAPHS CONSTRUCTED FROM LINEAR GROUPS $L(2, q)$ , $q \leq 23$

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ABSTRACT. In this paper we construct transitive  $t$ -designs from the linear groups  $L(2, q)$ ,  $q \leq 23$ . Thereby we classify  $t$ -designs,  $t \geq 2$ , admitting a transitive action of the linear groups  $L(2, q)$ ,  $q \leq 23$ , up to 35 points and obtained numerous transitive designs, for  $36 \leq v \leq 55$ . In many cases we proved the existence of  $t$ -designs with certain parameter sets. Among others we constructed  $t$ -designs with parameters  $2$ -(55, 10, 4),  $3$ -(24, 11, 495),  $3$ -(24, 12,  $5m$ ),  $m \in \{11, 12, 22, 33, 44, 66, 132\}$ . Furthermore, we constructed strongly regular graphs admitting a transitive action of the linear groups  $L(2, q)$ ,  $q \leq 23$ .

### 1. Introduction

We assume that the reader is familiar with the basic facts of group theory, design theory and theory of strongly regular graphs. We refer the reader to [2, 19], [7, 17] and [2, 3, 19] for related background materials in design theory, group theory and theory of strongly regular graphs, respectively.

An incidence structure is an ordered triple  $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$  where  $\mathcal{P}$  and  $\mathcal{B}$  are non-empty disjoint sets and  $\mathcal{I} \subseteq \mathcal{P} \times \mathcal{B}$ . The elements of the set  $\mathcal{P}$  are called points, the elements of the set  $\mathcal{B}$  are called blocks and  $\mathcal{I}$  is called an incidence relation. An incidence structure that does not contain repeated blocks (blocks incident with the same set of points) is called simple. If  $|\mathcal{P}| = |\mathcal{B}|$ , then the incidence structure is called symmetric. The incidence matrix of an incidence structure is a  $v \times b$  matrix  $[m_{ij}]$

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where  $v$  and  $b$  are the numbers of points and blocks respectively, such that  $m_{ij} = 1$  if the point  $P_i$  and the block  $x_j$  are incident, and  $m_{ij} = 0$  otherwise. An isomorphism from one incidence structure to another is a bijective mapping of points to points and blocks to blocks which preserves incidence. An isomorphism from an incidence structure  $\mathcal{D}$  onto itself is called an automorphism of  $\mathcal{D}$ . The set of all automorphisms forms a group called the full automorphism group of  $\mathcal{D}$  and is denoted by  $Aut(\mathcal{D})$ .

A  $t$ -( $v, k, \lambda$ ) design is a finite incidence structure  $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$  satisfying the following requirements:

- (1)  $|\mathcal{P}| = v$ ,
- (2) every element of  $\mathcal{B}$  is incident with exactly  $k$  elements of  $\mathcal{P}$ ,
- (3) every  $t$  elements of  $\mathcal{P}$  are incident with exactly  $\lambda$  elements of  $\mathcal{B}$ .

If  $\mathcal{D}$  is a  $t$ -design, then it is also an  $s$ -design, for  $1 \leq s \leq t - 1$ . A  $2$ -( $v, k, \lambda$ ) design is called a block design. We say that a  $t$ -( $v, k, \lambda$ ) design  $\mathcal{D}$  is a quasi-symmetric design with intersection numbers  $x$  and  $y$  ( $x < y$ ) if any two blocks of  $\mathcal{D}$  intersect in either  $x$  or  $y$  points.

A graph is regular if all the vertices have the same degree; a regular graph is strongly regular of type  $(v, k, \lambda, \mu)$  if it has  $v$  vertices, degree  $k$ , and if any two adjacent vertices are together adjacent to  $\lambda$  vertices, while any two non-adjacent vertices are together adjacent to  $\mu$  vertices. A strongly regular graph of type  $(v, k, \lambda, \mu)$  is usually denoted by  $SRG(v, k, \lambda, \mu)$ .

In this paper we consider  $t$ -designs and strongly regular graphs constructed from the linear groups  $L(2, q)$ ,  $q \leq 23$ .

The group  $L(2, q)$  acts 3-transitively on subsets of size 3 of the projective line if  $q \equiv 3 \pmod{4}$ . Thereby, unions of orbits of a subgroup of  $L(2, q)$  produce simple 3-designs under the action of  $L(2, q)$ . However, if  $q \equiv 1 \pmod{4}$ , the action is not 3-transitive so we do not have to obtain 3-designs. In [6], Cameron, Maimani, Omidi and Tayfeh-Rezaie determined all 3-designs admitting an automorphism group isomorphic to  $L(2, q)$  with block size not congruent to 0 and 1 modulo  $p$ , where  $q = p^n$  is a prime power congruent to 3 modulo 4. Further information on the research regarding linear groups  $L(2, q)$ , where  $q \equiv 1 \pmod{4}$ , can be found in [1], but the whole enumeration of  $t$ -designs admitting a transitive action of the groups  $L(2, q)$ ,  $q \leq 23$ , have not been described so far.

Using the method introduced in [10], we classify  $t$ -designs,  $t \geq 2$ , admitting a transitive action of the linear groups  $L(2, q)$ ,  $q \leq 23$ , up to 35 points. Additionally, we obtained numerous transitive designs, for  $36 \leq v \leq 55$ . All the designs obtained in this paper are simple. A number of the designs obtained in this paper including  $2$ -(55, 10, 4),  $3$ -(24, 11, 495),  $3$ -(24, 12,  $5m$ ),  $m \in \{11, 12, 22, 33, 44, 66, 132\}$  are the first known examples of the designs with these parameters. Further, we construct strongly regular graphs from the linear groups  $L(2, q)$ ,  $q \leq 23$ .

Generators of the linear groups  $L(2, q), q \leq 23$  are available via the Internet: <http://brauer.maths.qmul.ac.uk/Atlas/>. All the  $t$ -designs are obtained by using programmes written for Magma [5].

The paper is organized as follows: in Section 2 we briefly describe the method of construction of transitive designs and graphs used in this paper, and in Section 3 we describe  $t$ -designs and strongly regular graphs constructed under the action of the linear groups  $L(2, q), q \leq 23$ .

## 2. Designs and graphs constructed from groups

We say that an incidence structure  $\mathcal{I}$  is transitive if an automorphism group of  $\mathcal{I}$  acts transitively on points and blocks. An incidence structure  $\mathcal{I}$  is called primitive if an automorphism group acts primitively on points and blocks. Further, we say that graph  $\Gamma$  is transitive (primitive) if an automorphism group acts transitively (primitively) on the set of vertices of the graph  $\Gamma$ .

The construction of primitive symmetric 1-designs and regular graphs for which the stabilizer of a point and the stabilizer of a block are conjugate is presented in [11], [12] and [13]. The generalization, *i.e.* the method for constructing not necessarily symmetric but still primitive 1-designs, is presented in [8] and [9]. In [10], a construction of not necessarily primitive, but still transitive block designs is presented.

**Theorem 2.1** ([10]). *Let  $G$  be a finite permutation group acting transitively on the sets  $\Omega_1$  and  $\Omega_2$  of size  $m$  and  $n$ , respectively. Let  $\alpha \in \Omega_1$  and  $\Delta_2 = \bigcup_{i=1}^s \delta_i G_\alpha$ , where  $G_\alpha = \{g \in G \mid \alpha g = \alpha\}$  is the stabilizer of  $\alpha$  and  $\delta_1, \dots, \delta_s \in \Omega_2$  are representatives of distinct  $G_\alpha$ -orbits on  $\Omega_2$ . If  $\Delta_2 \neq \Omega_2$  and*

$$\mathcal{B} = \{\Delta_2 g : g \in G\},$$

*then  $\mathcal{D}(G, \alpha, \delta_1, \dots, \delta_s) = (\Omega_2, \mathcal{B})$  is a  $1-(n, |\Delta_2|, \frac{|G_\alpha|}{|G_{\Delta_2}} \sum_{i=1}^s |\alpha G_{\delta_i}|)$  design with  $\frac{m \cdot |G_\alpha|}{|G_{\Delta_2}|}$  blocks. The group  $H \cong G / \bigcap_{x \in \Omega_2} G_x$  acts as an automorphism group on  $(\Omega_2, \mathcal{B})$ , transitively on points and blocks of the design.*

*If  $\Delta_2 = \Omega_2$  then the set  $\mathcal{B}$  consists of one block, and  $\mathcal{D}(G, \alpha, \delta_1, \dots, \delta_s)$  is a design with parameters  $1-(n, n, 1)$ .*

If the group  $G$  acts  $t$ -transitively on the set  $\Omega_2$ , then the obtained design  $(\Omega_2, \mathcal{B})$  is a  $t$ -design (see [10]). However, if  $G$  does not act  $t$ -transitively on the set  $\Omega_2$ , the obtained design  $(\Omega_2, \mathcal{B})$  may still be a  $t$ -design.

In this paper we describe construction of  $t$ -designs, for  $t \geq 2$ , from linear groups  $L(2, q), q \leq 23$ , obtained by using Theorem 2.1.

**Corollary 2.2.** *If  $\Omega_1 = \Omega_2$  and  $\Delta_2$  is a union of self-paired and mutually paired orbits of  $G_\alpha$ , then the design  $\mathcal{D}(G, \alpha, \delta_1, \dots, \delta_s)$  is a symmetric self-dual design and the incidence matrix of that design is the adjacency matrix of a  $|\Delta_2|$ -regular graph.*

If  $G_\alpha = G_{\Delta_2}$ , we can interpret the design  $(\Omega_2, \mathcal{B})$  from Theorem 2.1 in the following way:

- the point set is  $\Omega_2$ ,
- the block set is  $\Omega_1 = \alpha G$ ,
- the block  $\alpha g'$  is incident with the set of points  $\{\delta_i g : g \in G_\alpha g', i = 1, \dots, s\} = \Delta_2 g'$ .

If  $G_\alpha \neq G_{\Delta_2}$ , then the above described design with the block set  $\Omega_1$  has repeated blocks, and the corresponding simple design is the design  $(\Omega_2, \mathcal{B})$  from Theorem 2.1, which has  $\frac{m \cdot |G_\alpha|}{|G_{\Delta_2}|}$  blocks.

Note that the construction from Theorem 2.1 gives us 1-designs and regular graphs. Since every  $t$ -design, for  $t \geq 1$ , is also a 1-design, Theorem 2.1 gives us all designs which admit a transitive action of the group  $G$  on points and blocks. Using this method of construction we obtained all 1-designs and regular graphs admitting a transitive action of the linear groups  $L(2, q), q \leq 23$  but we recorded only those 1-designs that are also  $t$ -designs,  $t \geq 2$ , and those regular graphs that are strongly regular.

### 3. Results

In this section we list all  $t$ -designs and strongly regular graphs that we constructed using the method described in Section 2. Each subsection contains the information about the linear group that we used, subgroups of the group and its properties and the data about obtained  $t$ -designs. The last section contains information about constructed strongly regular graphs. For each constructed structure ( $t$ -design and strongly regular graph) we give the structure of the full automorphism group, the number of non-isomorphic  $t$ -designs and remarks about constructed  $t$ -design and SRG. In tables to follow, for designs indicated by \* we proved the existence in terms of parameters.

**3.1. Transitive  $t$ -designs from  $L(2, 5)$ .** It is well known (see [7]) that the group  $L(2, 5) \cong L(2, 4) \cong A_5$ . The linear group  $L(2, 5)$  is the simple group of order 60 and up to conjugation it has 9 subgroups. In Table 1 we give the list of all the subgroups, up to conjugation. We classify  $t$ -designs,  $t \geq 2$ , admitting transitive action of the group  $L(2, 5)$ . The constructed designs are presented in Table 2.

TABLE 1. Subgroups of the group  $L(2, 5)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
$I$	1	60	No	$S_3$	6	10	Yes
$Z_2$	2	30	No	$D_{10}$	10	6	Yes
$Z_3$	3	20	No	$A_4$	12	5	Yes
$E_4$	4	15	No	$A_5$	60	1	No
$Z_5$	5	12	No				

TABLE 2.  $t$ -designs constructed from the group  $L(2, 5)$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-(6, 3, 2)	10	1	$A_5$
2-(10, 3, 2)	30	1	$A_5$
2-(10, 4, 2)	15	1	$S_6$
2-(10, 4, 8)	60	1	$S_5$
2-(15, 7, 3)	15	1	$A_8$
2-(15, 7, 12)	60	1	$GL(2, 4) : Z_2$
		2	$GL(2, 4)$

**Remark 3.1.** *Transitive  $t$ -designs described in Table 2 were all previously known. For further information on quasi-symmetric 2-(10, 4, 2) we refer the reader to [18] and for other designs we refer the reader to [16].*

**3.2. Transitive  $t$ -designs from  $L(2, 7)$ .** The linear group  $L(2, 7)$  is the simple group of order 168 and up to conjugation it has 15 subgroups. It is well known (see [7]) that  $L(2, 7) \cong L(3, 2)$ . We classify  $t$ -designs,  $t \geq 2$ ,  $v < 56$  admitting transitive action of the group  $L(2, 7)$ . In Table 3 we give the list of all the subgroups, up to conjugation, and in Table 4 we present the constructed designs.

TABLE 3. Subgroups of the group  $L(2, 7)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
$I$	1	168	No	$D_8$	8	21	No
$Z_2$	2	84	No	$A_4$	12	14	No
$Z_3$	3	56	No	$A_4$	12	14	No
$E_4$	4	42	No	$Z_7 : Z_3$	21	8	Yes
$E_4$	4	42	No	$S_4$	24	7	Yes
$Z_4$	4	42	No	$S_4$	24	7	Yes
$S_3$	6	28	No	$L(2, 7)$	168	1	No
$Z_7$	7	24	No				

TABLE 4.  $t$ -designs constructed from the group  $L(2, 7)$ ,  $v < 56$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-(7, 3, 1)	7	1	$L(2, 7)$
2-(7, 3, 4)	28	1	$L(2, 7)$
3-(8, 3, 1)	56	1	$S_8$
3-(8, 4, 1)	14	1	$E_8 : L(2, 7)$
3-(8, 4, 3)	42	1	$L(2, 7) : Z_2$
2-(21, 5, 4)	84	2	$L(2, 7)$
2-(28, 9, 16)*	168	85	$L(2, 7)$
		5	$L(2, 7)$
2-(28, 10, 20)*	168	1	$E_8 : L(2, 7)$
		8	$L(2, 7) : Z_2$
		213	$L(2, 7)$

**Remark 3.2.** Designs with parameters 2-(28, 9, 16) and 2-(28, 10, 20) are not mentioned in the [16] since  $r > 41$ . Up to our best knowledge they have not been known before, so we proved the existence of the 2-(28, 9, 16) and 2-(28, 10, 20). All others transitive  $t$ -designs described in Table 4 were previously known (see [14, 16]). For further information on unique quasi-symmetric 3-(8, 4, 1) design we refer the reader to [18].

**3.3. Transitive  $t$ -designs from  $L(2, 8)$ .** The linear group  $L(2, 8)$  is the simple group of order 504 and up to conjugation it has 12 subgroups, given in Table 5. We classify  $t$ -designs,  $t \geq 2$ ,  $v \leq 28$  admitting transitive action of the group  $L(2, 8)$  and the obtained designs are listed in Table 6. Additionally, we classify  $t$ -designs,  $t \geq 2$ ,  $v = 36$ ,  $b \leq 252$  admitting transitive action of the group  $L(2, 8)$  and the obtained designs are listed in Table 7.

TABLE 5. Subgroups of the group  $L(2, 8)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
$I$	1	504	No	$E_8$	8	63	No
$Z_2$	2	252	No	$Z_9$	9	56	No
$Z_3$	3	168	No	$D_{14}$	14	36	Yes
$E_4$	4	126	No	$D_{18}$	18	28	Yes
$S_3$	6	84	No	$E_8 : Z_7$	56	9	Yes
$Z_7$	7	72	No	$L(2, 8)$	504	1	No

TABLE 6.  $t$ -designs constructed from the group  $L(2, 8)$ ,  $v \leq 28$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
4-(9, 4, 1)	126	1	$S_9$
3-(9, 3, 1)	84	1	$S_9$
2-(28, 3, 4)*	504	1	$L(2, 8) : Z_3$
2-(28, 4, 1)	63	1	$L(2, 8) : Z_3$
2-(28, 4, 8)*	504	1	$L(2, 8) : Z_3$
		1	$L(2, 8)$
2-(28, 6, 20)*	504	3	$L(2, 8) : Z_3$
		16	$L(2, 8)$
2-(28, 7, 14)*	252	1	$L(2, 8)$
2-(28, 7, 28)*	504	4	$L(2, 8) : Z_3$
		12	$L(2, 8)$
2-(28, 9, 48)*	504	12	$L(2, 8) : Z_3$
		158	$L(2, 8)$
2-(28, 10, 30)*	252	2	$L(2, 8)$
2-(28, 10, 60)*	504	11	$L(2, 8) : Z_3$
		290	$L(2, 8)$
2-(28, 12, 11)	63	1	$L(2, 8)$
		1	$S(6, 2)$
2-(28, 12, 44)*	252	1	$L(2, 8)$
2-(28, 12, 88)*	504	19	$L(2, 8) : Z_3$
		587	$L(2, 8)$
2-(28, 13, 52)*	252	2	$L(2, 8)$
2-(28, 13, 104)*	504	18	$L(2, 8) : Z_3$
		723	$L(2, 8)$

TABLE 7.  $t$ -designs constructed from the group  $L(2, 8)$ ,  $v = 36, b \leq 252$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-(36, 5, 4)	252	1	$L(2, 8)$
2-(36, 6, 2)	84	1	$L(2, 8) : Z_3$
2-(36, 6, 6)*	252	3	$L(2, 8) : Z_3$
2-(36, 10, 9)	126	1	$L(2, 8) : Z_3$
2-(36, 10, 18)*	252	4	$L(2, 8) : Z_3$
		1	$L(2, 8)$
2-(36, 11, 22)*	252	4	$L(2, 8) : Z_3$
2-(36, 15, 28)*	168	1	$L(2, 8) : Z_3$
		1	$L(2, 8)$
2-(36, 16, 12)	63	1	$L(2, 8) : Z_3$
		1	$S(6, 2)$
2-(36, 16, 48)*	252	2	$L(2, 8) : Z_3$

**Remark 3.3.** For  $t = 3, 4$  designs described in Table 6 were known before. For quasi-symmetric 2-(28, 4, 1), 2-(28, 12, 11) and 2-(36, 16, 12) designs we refer the reader to [10, 16, 18]. Majority of 2-designs listed in Table 6 and Table 7 have  $r > 41$ .

3.4. **Transitive  $t$ -designs from  $L(2, 9)$ .** It is well known (see [7]) that  $L(2, 9) \cong S(4, 2)' \cong A_6 \cong M'_{10}$ . The linear group  $L(2, 9)$  is the simple group of order 360 and up to conjugation it has 22 subgroups, listed in Table 8. We classify  $t$ -designs,  $t \geq 2, v \leq 30$  admitting transitive action of the group  $L(2, 8)$  and the obtained designs are listed in Table 9. Additionally, we classify  $t$ -designs,  $t \geq 2, v = 36, b \leq 180$  and  $v = 40, b \leq 180$  admitting transitive action of the group  $L(2, 9)$  and the obtained designs are listed in Table 10.

TABLE 8. Subgroups of the group  $L(2, 9)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
$I$	1	360	No	$E_9$	9	40	No
$Z_2$	2	180	No	$D_{10}$	10	36	No
$Z_3$	3	120	No	$A_4$	12	30	No
$Z_3$	3	120	No	$A_4$	12	30	No
$E_4$	4	90	No	$E_9 : Z_2$	18	20	No
$E_4$	4	90	No	$S_4$	24	15	Yes
$Z_4$	4	90	No	$S_4$	24	15	Yes
$Z_5$	5	72	No	$E_9 : Z_4$	36	10	Yes
$S_3$	6	60	No	$A_5$	60	6	Yes
$S_3$	6	60	No	$A_5$	60	6	Yes
$D_8$	8	45	No	$L(2, 9)$	360	1	No



TABLE 9.  $t$ -designs constructed from the group  $L(2, 9)$ ,  $v \leq 30$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
3-(6, 3, 1)	20	1	$S_6$
3-(10, 4, 6)	180	1	$A_6 : E_4$
3-(10, 5, 3)	36	1	$A_6 \cdot Z_2$
2-(10, 3, 4)	60	1	$S_6$
2-(10, 4, 2)	15	1	$S_6$
2-(10, 5, 20)*	90	1	$S_6$
2-(15, 7, 3)	15	1	$A_8$
2-(15, 7, 24)*	120	1	$A_8$
2-(15, 7, 36)*	180	1	$S_6$
2-(15, 7, 72)*	360	1	$A_6$
		2	$S_6$

TABLE 10.  $t$ -designs constructed from the group  $L(2, 9)$ ,  $v = 36, 40, b \leq 180$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-(36, 8, 4)	90	1	$S_6$
2-(36, 15, 12)	72	1	$A_6$
		1	$S_6$
2-(36, 15, 30)*	180	3	$A_6$
2-(40, 13, 4)	40	1	$L(4, 3) : Z_2$

**Remark 3.4.** For  $t = 3$  designs described in Table 9 were known before.

**3.5. Transitive  $t$ -designs from  $L(2, 11)$ .** The linear group  $L(2, 11)$  is the simple group of order 660 and up to conjugation it has 16 subgroups, which are listed in Table 11. In Table 12 we give the information on the obtained designs.

We classify  $t$ -designs,  $t \geq 2$ ,  $v < 55$  admitting transitive action of the group  $L(2, 11)$  and the obtained designs are listed in Table 12. Additionally, we classify  $t$ -designs,  $t \geq 2$ ,  $v = 55$ ,  $b \leq 330$ , admitting transitive action of the group  $L(2, 11)$  and the obtained designs are listed in Table 13.

TABLE 11. Subgroups of the group  $L(2, 11)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
$I$	1	660	No	$D_{10}$	10	66	No
$Z_2$	2	330	No	$Z_{11}$	11	60	No
$Z_3$	3	220	No	$A_4$	12	55	No
$E_4$	4	165	No	$D_{12}$	12	55	Yes
$Z_5$	5	132	No	$Z_{11} : Z_5$	55	12	Yes
$S_3$	6	110	No	$A_5$	60	11	Yes
$S_3$	6	110	No	$A_5$	60	11	Yes
$Z_6$	6	110	No	$L(2, 11)$	660	1	No

TABLE 12.  $t$ -designs constructed from the group  $L(2, 11)$ ,  $v < 55$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-(11, 3, 3)	55	1	$L(2, 11)$
2-(11, 3, 6)	110	1	$L(2, 11)$
2-(11, 4, 6)	55	1	$L(2, 11)$
2-(11, 4, 12)	110	1	$L(2, 11)$
2-(11, 4, 18)*	165	1	$L(2, 11)$
2-(11, 5, 2)	11	1	$L(2, 11)$
2-(11, 5, 10)	55	1	$L(2, 11)$
2-(11, 5, 12)	66	1	$L(2, 11)$
2-(11, 5, 60)*	330	1	$L(2, 11)$
5-(12, 6, 1)	132	1	$M_{12}$
3-(12, 3, 1)	220	1	$S_{12}$
3-(12, 4, 3)	165	1	$L(2, 11) : Z_2$
3-(12, 4, 6)	330	1	$L(2, 11) : Z_2$
3-(12, 5, 6)	132	1	$L(2, 11) : Z_2$
3-(12, 5, 30)	660	1	$L(2, 11) : Z_2$
3-(12, 6, 10)	110	1	$L(2, 11) : Z_2$
		1	$L(2, 11)$
3-(12, 6, 30)	330	1	$L(2, 11) : Z_2$

TABLE 13.  $t$ -designs constructed from the group  $L(2, 11)$ ,  $v = 55, b \leq 330$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
$2-(55, 9, 8)^*$	330	2	$L(2, 11)$
$2-(55, 10, 4)^*$	132	2	$L(2, 11)$
$2-(55, 10, 10)^*$	330	6	$L(2, 11)$
$2-(55, 18, 34)^*$	330	1	$L(2, 11) : Z_2$
		29	$L(2, 11)$
$2-(55, 19, 38)^*$	330	7	$L(2, 11)$
$2-(55, 27, 78)^*$	330	1	$L(2, 11) : Z_2$
		48	$L(2, 11)$

**Remark 3.5.** For  $t > 2$  we refer the reader to [14]. The design  $5-(12, 6, 1)$  is famous Witt design,  $W_{12}$ , on which the Mathieu group  $M_{12}$  acts 5-transitively. For further information on  $W_{12}$  we refer the reader to [20, 21].

**3.6. Transitive  $t$ -designs from  $L(2, 13)$ .** The linear group  $L(2, 13)$  is the simple group of order 1092 and up to conjugation it has 16 subgroups. We classify  $t$ -designs,  $t \geq 2, v < 78$  admitting transitive action of the group  $L(2, 13)$ . These subgroups are listed in Table 14, and in Table 15 we give the information on the obtained designs.

TABLE 14. Subgroups of the group  $L(2, 13)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
$I$	1	1092	No	$D_{12}$	12	91	Yes
$Z_2$	2	546	No	$A_4$	12	91	Yes
$Z_3$	3	364	No	$Z_{13}$	13	84	No
$E_4$	4	273	No	$D_{14}$	14	78	Yes
$S_3$	6	182	No	$D_{26}$	26	42	No
$S_3$	6	182	No	$Z_{13} : Z_3$	39	28	No
$Z_6$	6	182	No	$Z_{13} : Z_6$	78	14	Yes
$Z_7$	7	156	No	$L(2, 13)$	1092	1	No

TABLE 15.  $t$ -designs constructed from the group  $L(2, 13)$ ,  $v < 78$ 

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
3-(14, 4, 3)	273	1	$L(2, 13) : Z_2$
3-(14, 4, 6)	546	1	$L(2, 13) : Z_2$
3-(14, 5, 15)	546	1	$L(2, 13) : Z_2$
3-(14, 6, 5)	91	3	$L(2, 13) : Z_2$
3-(14, 6, 30)	546	1	$L(2, 13) : Z_2$
3-(14, 6, 60)	1092	1	$L(2, 13) : Z_2$
2-(14, 3, 6)	182	1	$L(2, 13)$
2-(14, 4, 6)	91	1	$L(2, 13)$
2-(14, 5, 20)*	182	1	$L(2, 13)$
2-(14, 5, 60)*	546	1	$L(2, 13)$
2-(14, 6, 15)	91	1	$L(2, 13)$
2-(14, 6, 90)*	546	1	$L(2, 13)$
2-(14, 7, 18)	78	1	$L(2, 13)$
2-(14, 7, 42)*	182	1	$L(2, 13)$
2-(14, 7, 84)*	364	1	$L(2, 13)$
2-(14, 7, 126)*	546	2	$L(2, 13)$

**Remark 3.6.** For  $t = 3$  we refer the reader to [14].

**3.7. Transitive  $t$ -designs from  $L(2, 16)$ .** The linear group  $L(2, 16)$  is the simple group of order 4080 and up to conjugation it has 21 subgroups, listed in Table 16. We classify  $t$ -designs,  $t \geq 2$ ,  $v < 51$  admitting transitive action of the group  $L(2, 16)$ . The constructed designs are described in Table 17.

Additionally, we classify  $t$ -designs,  $t \geq 2$ ,  $v = 51$ ,  $b \leq 2040$  admitting transitive action of the group  $L(2, 16)$  and the obtained designs are listed in Table 18.

TABLE 16. Subgroups of the group  $L(2, 16)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
$I$	1	4080	No	$Z_{15}$	15	272	No
$Z_2$	2	2040	No	$E_{16}$	16	255	No
$Z_3$	3	1360	No	$Z_{17}$	17	240	No
$E_4$	4	1020	No	$D_{30}$	30	136	Yes
$E_4$	4	1020	No	$D_{34}$	34	120	Yes
$E_4$	4	1020	No	$E_{16} : Z_3$	48	85	No
$Z_5$	5	816	No	$A_5$	60	68	Yes
$S_3$	6	680	No	$E_{16} : Z_5$	80	51	No
$E_8$	8	510	No	$E_{16} : Z_{15}$	240	17	Yes
$D_{10}$	10	408	No	$L(2, 16)$	4080	1	No
$A_4$	12	340	No				

TABLE 17.  $t$ -designs constructed from the group  $L(2, 16)$ ,  $v < 51$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
4-(17, 7, 6)	408	1	$L(2, 16) : Z_4$
4-(17, 7, 60)	4080	1	$L(2, 16) : Z_4$
4-(17, 8, 15)	510	1	$L(2, 16) : Z_4$
4-(17, 8, 120)	4080	1	$L(2, 16) : Z_4$
3-(17, 3, 1)	680	1	$S_{17}$
3-(17, 4, 2)	340	1	$L(2, 16) : Z_4$
3-(17, 4, 6)	1020	1	$L(2, 16) : Z_2$
3-(17, 5, 1)	68	1	$L(2, 16) : Z_4$
3-(17, 5, 15)	1020	1	$L(2, 16) : Z_2$
3-(17, 5, 60)	4080	1	$L(2, 16) : Z_4$
3-(17, 6, 20)	680	1	$L(2, 16) : Z_2$
3-(17, 6, 24)	816	1	$L(2, 16) : Z_4$
3-(17, 6, 60)	2040	1	$L(2, 16) : Z_4$
		1	$L(2, 16)$
3-(17, 7, 70)	1360	1	$L(2, 16) : Z_2$
3-(17, 7, 105)	2040	2	$L(2, 16)$
3-(17, 8, 56)	680	2	$L(2, 16) : Z_2$
3-(17, 8, 168)	2040	1	$L(2, 16) : Z_4$
		1	$L(2, 16)$
3-(17, 8, 336)	4080	1	$L(2, 16) : Z_2$

TABLE 18.  $t$ -designs constructed from the group  $L(2, 16)$ ,  $v = 51, b \leq 2040$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-(51, 25, 240)*	1020	2	$(Z_3 \times L(2, 16)) : Z_4$
		2	$Z_3 \times (L(2, 16) : Z_2)$
2-(51, 25, 480)*	2040	117	$L(2, 16)$
		4	$L(2, 16) : Z_2$

**Remark 3.7.** For designs with  $t = 3, 4$  we refer the reader to [14].

**3.8. Transitive  $t$ -designs from  $L(2, 17)$ .** The linear group  $L(2, 17)$  is the simple group of order 2448 and up to conjugation it has 22 subgroups. The subgroups are listed in Table 19, and the constructed designs are given in Table 20. We classify  $t$ -designs,  $t \geq 2, v < 36$  admitting transitive action of the group  $L(2, 17)$  and the obtained designs are listed in Table 20. Additionally, we classify  $t$ -designs,  $t \geq 2, v = 36, b \leq 1224$  admitting transitive action of the group  $L(2, 17)$  and the obtained designs are listed in Table 21.

TABLE 19. Subgroups of the group  $L(2, 17)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
$I$	1	2448	No	$A_4$	12	204	No
$Z_2$	2	1224	No	$A_4$	12	204	No
$Z_3$	3	816	No	$D_{16}$	16	153	Yes
$E_4$	4	612	No	$Z_{17}$	17	144	No
$E_4$	4	612	No	$D_{18}$	18	136	Yes
$Z_4$	4	612	No	$S_4$	24	102	Yes
$S_3$	6	408	No	$S_4$	24	102	Yes
$D_8$	8	306	No	$D_{34}$	34	72	No
$D_8$	8	306	No	$Z_{17} : Z_4$	68	36	No
$Z_8$	8	306	No	$Z_{17} : Z_8$	136	18	Yes
$Z_9$	9	272	No	$L(2, 17)$	2448	1	No

TABLE 20.  $t$ -designs constructed from the group  $L(2, 17)$ ,  $v < 36$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
5-(18, 8, 16)	2448	1	$L(2, 17) : Z_2$
3-(18, 4, 6)	1224	2	$L(2, 17) : Z_2$
3-(18, 5, 15)	1224	1	$L(2, 17)$
3-(18, 6, 10)	408	1	$L(2, 17) : Z_2$
3-(18, 6, 20)	816	1	$L(2, 17) : Z_2$
3-(18, 6, 30)	1224	1	$L(2, 17) : Z_2$
3-(18, 6, 60)	2448	2	$L(2, 17) : Z_2$
3-(18, 8, 42)	612	1	$L(2, 17) : Z_2$
		2	$L(2, 17)$
3-(18, 8, 84)	1224	2	$L(2, 17) : Z_2$
		2	$L(2, 17)$
3-(18, 8, 168)	2448	1	$L(2, 17) : Z_2$
3-(18, 9, 252)	2448	1	$L(2, 17) : Z_2$
2-(18, 3, 8)*	408	1	$L(2, 17)$
2-(18, 4, 12)*	306	2	$L(2, 17)$
2-(18, 5, 40)*	612	1	$L(2, 17)$
2-(18, 5, 80)*	1224	2	$L(2, 17)$
2-(18, 6, 10)	102	1	$L(2, 17)$
2-(18, 6, 60)*	612	1	$L(2, 17)$
2-(18, 6, 120)*	1224	2	$L(2, 17)$
2-(18, 6, 240)*	2448	1	$L(2, 17)$
2-(18, 7, 168)*	1224	11	$L(2, 17)$
2-(18, 7, 336)*	2448	3	$L(2, 17)$
2-(18, 8, 28)*	153	1	$L(2, 17)$
2-(18, 8, 224)*	1224	2	$L(2, 17)$
2-(18, 8, 448)*	2448	4	$L(2, 17)$
2-(18, 9, 32)*	136	2	$L(2, 17)$
2-(18, 9, 72)*	306	1	$L(2, 17)$
2-(18, 9, 96)*	408	2	$L(2, 17)$
2-(18, 9, 144)*	612	1	$L(2, 17)$
2-(18, 9, 192)*	816	1	$L(2, 17)$
2-(18, 9, 288)*	1224	9	$L(2, 17)$
2-(18, 9, 576)*	2448	5	$L(2, 17)$

TABLE 21.  $t$ -designs constructed from the group  $L(2, 17)$ ,  $v = 36, b \leq 1224$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
$2-(36, 15, 68)^*$	408	3	$L(2, 17)$
$2-(36, 15, 136)^*$	816	7	$L(2, 17)$
$2-(36, 15, 204)^*$	1224	57	$L(2, 17)$

**Remark 3.8.** For information on known 3-designs with 18 points the reader may consult [14].

**3.9. Transitive  $t$ -designs from  $L(2, 19)$ .** The linear group  $L(2, 19)$  is the simple group of order 3420 and up to conjugation it has 19 subgroups. We classify  $t$ -designs,  $t \geq 2, v < 57$  admitting transitive action of the group  $L(2, 19)$ . These subgroups are listed in Table 22, and the information on constructed designs are given in Table 23.

TABLE 22. Subgroups of the group  $L(2, 19)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
$I$	1	3420	No	$A_4$	12	285	No
$Z_2$	2	1710	No	$D_{18}$	18	190	Yes
$Z_3$	3	1140	No	$Z_{19}$	19	180	No
$E_4$	4	855	No	$D_{20}$	20	171	Yes
$Z_5$	5	684	No	$Z_{19} : Z_3$	57	60	No
$S_3$	6	570	No	$A_5$	60	57	Yes
$Z_9$	9	380	No	$A_5$	60	57	Yes
$D_{10}$	10	342	No	$Z_{19} : Z_9$	171	20	Yes
$D_{10}$	10	342	No	$L(2, 19)$	3420	1	No
$Z_{10}$	10	342	No				



TABLE 23.  $t$ -designs constructed from the group  $L(2, 19)$ ,  $v < 57$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
3-(20, 3, 1)	1140	1	$S_{20}$
3-(20, 4, 1)	285	1	$L(2, 19)$
3-(20, 4, 3)	855	1	$L(2, 19) : Z_2$
3-(20, 4, 6)	1710	2	$L(2, 19) : Z_2$
3-(20, 5, 6)	684	1	$L(2, 19) : Z_2$
3-(20, 5, 10)	1140	1	$L(2, 19) : Z_2$
3-(20, 5, 30)	3420	2	$L(2, 19) : Z_2$
		1	$L(2, 19)$
3-(20, 6, 10)	570	1	$L(2, 19) : Z_2$
		1	$L(2, 19)$
3-(20, 6, 20)	1140	1	$L(2, 19) : Z_2$
3-(20, 6, 30)	1710	3	$L(2, 19) : Z_2$
		3	$L(2, 19)$
3-(20, 6, 60)	3420	2	$L(2, 19) : Z_2$
		2	$L(2, 19)$
3-(20, 7, 35)	1140	1	$L(2, 19) : Z_2$
		2	$L(2, 19)$
3-(20, 7, 105)	3420	9	$L(2, 19) : Z_2$
		6	$L(2, 19)$
3-(20, 8, 14)	285	1	$L(2, 19) : Z_2$
3-(20, 8, 28)	570	1	$L(2, 19) : Z_2$
		1	$L(2, 19)$
3-(20, 8, 42)	855	1	$L(2, 19) : Z_2$
		1	$L(2, 19)$
3-(20, 8, 84)	1710	3	$L(2, 19) : Z_2$
		6	$L(2, 19)$
3-(20, 8, 168)	3420	7	$L(2, 19) : Z_2$
		11	$L(2, 19)$
3-(20, 9, 28)	380	1	$L(2, 19) : Z_2$
3-(20, 9, 84)	1140	1	$L(2, 19) : Z_2$
		1	$L(2, 19)$
3-(20, 9, 252)	3420	12	$L(2, 19) : Z_2$
		21	$L(2, 19)$
3-(20, 10, 36)	342	1	$L(2, 19) : Z_2$
		1	$L(2, 19)$
3-(20, 10, 40)	380	1	$L(2, 19)$
3-(20, 10, 120)	1140	3	$L(2, 19)$
3-(20, 10, 180)	1710	5	$L(2, 19) : Z_2$
		10	$L(2, 19)$
3-(20, 10, 360)	3420	8	$L(2, 19) : Z_2$
		16	$L(2, 19)$

**Remark 3.9.** For information on known 3-designs with 20 points the reader may consult [14].

3.10. **Transitive  $t$ -designs from  $L(2, 23)$ .** The linear group  $L(2, 23)$  is the simple group of order 6072 and up to conjugation it has 23 subgroups, which are listed in Table 24. We classify  $t$ -designs,  $t \geq 2$ ,  $v < 253$  admitting transitive action of the group  $L(2, 23)$ . The information on constructed designs are given in Tables 25 and 26.

TABLE 24. Subgroups of the group  $L(2, 23)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
$I$	1	6072	No	$A_4$	12	506	No
$Z_2$	2	3036	No	$D_{12}$	12	506	No
$Z_3$	3	2024	No	$D_{12}$	12	506	No
$E_4$	4	1518	No	$A_4$	12	506	No
$E_4$	4	1518	No	$D_{22}$	22	276	Yes
$Z_4$	4	1518	No	$Z_{23}$	23	264	No
$Z_6$	6	1518	No	$S_4$	24	253	Yes
$S_3$	6	1012	No	$S_4$	24	253	Yes
$S_3$	6	1012	No	$D_{24}$	24	253	Yes
$D_8$	8	759	No	$Z_{23} : Z_{11}$	253	24	Yes
$Z_{11}$	11	552	No	$L(2, 23)$	6072	1	No
$Z_{12}$	12	506	No				

TABLE 25. 5-designs constructed from the group  $L(2, 23)$ ,  $v < 253$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
5-(24, 7, 3)	6072	1	$L(2, 23)$
5-(24, 8, 1)	759	1	$M_{24}$
5-(24, 9, 6)	2024	1	$L(2, 23)$
5-(24, 10, 36)	6072	1	$L(2, 23)$

TABLE 26. 3-designs constructed from the group  $L(2, 23)$ ,  $v < 253$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
3-(24, 3, 1)	2024	1	$S_{24}$
3-(24, 4, 3)	1518	1	$L(2, 23) : Z_2$
		1	$L(2, 23)$
3-(24, 4, 6)	3036	2	$L(2, 23) : Z_2$
3-(24, 5, 30)	6072	5	$L(2, 23) : Z_2$
		1	$L(2, 23)$
3-(24, 6, 10)	1012	1	$L(2, 23) : Z_2$
		2	$L(2, 23)$
3-(24, 6, 20)	2024	1	$L(2, 23) : Z_2$
3-(24, 6, 30)	3036	4	$L(2, 23) : Z_2$
		5	$L(2, 23)$
3-(24, 6, 60)	6072	4	$L(2, 23) : Z_2$
		5	$L(2, 23)$
3-(24, 7, 105)	6072	15	$L(2, 23) : Z_2$
		20	$L(2, 23)$
3-(24, 8, 21)	759	1	$L(2, 23) : Z_2$
3-(24, 8, 42)	1518	1	$L(2, 23) : Z_2$
		1	$L(2, 23)$
3-(24, 8, 84)	3036	6	$L(2, 23) : Z_2$
		14	$L(2, 23)$
3-(24, 8, 168)	6072	15	$L(2, 23) : Z_2$
		44	$L(2, 23)$
3-(24, 9, 84)	2024	3	$L(2, 23) : Z_2$
		1	$L(2, 23)$
3-(24, 9, 252)	6072	27	$L(2, 23) : Z_2$
		93	$L(2, 23)$
3-(24, 10, 180)	3036	10	$L(2, 23) : Z_2$
		28	$L(2, 23)$
3-(24, 10, 360)	6072	24	$L(2, 23) : Z_2$
		132	$L(2, 23)$
3-(24, 11, 45)	552	1	$L(2, 23) : Z_2$
3-(24, 11, 495)	6072	41	$L(2, 23) : Z_2$
		185	$L(2, 23)$
3-(24, 12, 55)	506	1	$L(2, 23) : Z_2$
		2	$L(2, 23)$
3-(24, 12, 60)	552	2	$L(2, 23)$
3-(24, 12, 110)	1012	2	$L(2, 23)$
3-(24, 12, 165)	1518	1	$L(2, 23) : Z_2$
		4	$L(2, 23)$
3-(24, 12, 220)	2024	2	$L(2, 23) : Z_2$
		1	$L(2, 23)$
3-(24, 12, 330)	3036	8	$L(2, 23) : Z_2$
		27	$L(2, 23)$
3-(24, 12, 660)	6072	30	$L(2, 23) : Z_2$
		190	$L(2, 23)$

**Remark 3.10.** In [6] Cameron determined parameters for possible 3-designs obtained from the group  $L(2, q)$ ,  $q \equiv 3 \pmod{4}$ , but there the 3-designs have not been enumerated. In [14, 15], the designs with parameters  $3$ -(24, 11, 495) and  $3$ -(24, 12,  $5m$ ),  $m \in \{11, 12, 22, 33, 44, 66, 132\}$  from Table 26 are mentioned as unknown. The design  $5$ -(24, 8, 1) is famous Witt design,  $W_{24}$ , on which the Mathieu group  $M_{24}$  acts 5-transitively. For further information on  $W_{24}$  we refer the reader to [20, 21].

**3.11. SRGs from  $L(2, q)$ ,  $q \leq 23$ .** Using the method described in Theorem 2.1 and Corollary 2.2, we obtained regular graphs on which the linear groups  $L(2, q)$ ,  $q \leq 23$  act transitively. Using the computer search we obtained strongly regular graphs. Finally, we determined the full automorphism groups of the constructed SRGs. In Table 27 we give the details about obtained graphs i.e. the linear group which acts transitively on the obtained graph, the parameters and the full automorphism group of the graph.

TABLE 27. SRGs from  $L(2, q)$ ,  $q \leq 23$ 

Graph $\Gamma$	Parameters	$Aut(\Gamma)$
$\Gamma_1 = \Gamma(L(2, 5))$	(10,3,0,1)	$S_5$
$\Gamma_2 = \Gamma(L(2, 5))$	(15,6,1,3)	$S_6$
$\Gamma_3 = \Gamma(L(2, 7))$	(21,10,3,6)	$S_7$
$\Gamma_4 = \Gamma(L(2, 7))$	(28,12,6,4)	$S_8$
$\Gamma_5 = \Gamma(L(2, 8))$	(36,14,7,4)	$S_9$
$\Gamma_6 = \Gamma(L(2, 8))$	(63,30,13,15)	$L(2, 8) : Z_3$
$\Gamma_7 = \Gamma(L(2, 8))$	(63,30,13,15)	$O(7, 2)$
$\Gamma_8 = \Gamma(L(2, 9))$	(15,6,1,3)	$S_6$
$\Gamma_9 = \Gamma(L(2, 9))$	(36,10,4,2)	$(S_6 \times S_6) : Z_2$
$\Gamma_{10} = \Gamma(L(2, 9))$	(40,12,2,4)	$O(5, 3) : Z_2$
$\Gamma_{11} = \Gamma(L(2, 9))$	(45,16,8,4)	$S_{10}$
$\Gamma_{12} = \Gamma(L(2, 11))$	(55,18,9,4)	$S_{11}$
$\Gamma_{13} = \Gamma(L(2, 11))$	(66,20,10,4)	$S_{12}$
$\Gamma_{14} = \Gamma(L(2, 13))$	(91,24,12,4)	$S_{14}$
$\Gamma_{15} = \Gamma(L(2, 16))$	(120,51,18,24)	$O(5, 4) : Z_2$
$\Gamma_{16} = \Gamma(L(2, 16))$	(136,30,15,4)	$S_{17}$
$\Gamma_{17} = \Gamma(L(2, 16))$	(136,60,24,28)	$O(5, 4) : Z_2$
$\Gamma_{18} = \Gamma(L(2, 17))$	(136,63,30,28)	$L(2, 17)$
$\Gamma_{19} = \Gamma(L(2, 17))$	(153,32,16,4)	$S_{18}$
$\Gamma_{20} = \Gamma(L(2, 19))$	(190,36,18,4)	$S_{20}$

The SRGs  $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \Gamma_8, \Gamma_{11}, \Gamma_{12}, \Gamma_{13}, \Gamma_{14}, \Gamma_{16}, \Gamma_{19}, \Gamma_{20}$ , are triangular graphs. The graph  $\Gamma_9$  is the unique graph with this parameters. The graph  $\Gamma_{10}$  and  $\Gamma_7$  are  $O(5, 3)$  and  $O(7, 2)$  graphs, respectively, they are obtained by taking the points on a nondegenerate quadric in  $PG(4,3)$  and

$PG(6,2)$ , respectively, and defining the adjacency by orthogonality. The graphs  $\Gamma_{15}$  and  $\Gamma_{18}$  are polar graphs  $NO^-(5,4)$  and  $NO^-(8,2)$  respectively, while  $\Gamma_{17}$  is the complement of the polar graph  $NO^+(5,4)$ . For further information we refer the reader to [3, 4].

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